Adult Literacy Fundamental Mathematics: Book 6 - 2nd Edition

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To the Learner

Welcome to *Adult Literacy Fundamental Mathematics: Book 6*.

You have the skills you need to be a strong student in this class. Your instructor knows this because you have passed the Adult Literacy Fundamental Mathematics Level 5 class, or you have been assessed into this level.

Adult math learners have many skills. They have a lot of life experience. They also use math in their everyday lives. This means that adult math learners may already know some of what is being taught in this book. Use what you already know with confidence!

How to Use This Book

This textbook has:

- A **Table of Contents** listing the units, the major topics, and the subtopics.
- A **Grades Record** to keep track of your marks.
- Many **Exercises** to practice what you learned. Some are quite short, but others have a great number of questions. You do not have to do every single question!
    - Do as many questions as you feel are necessary for you to be confident in your skill. It is best to do all the word problems.
    - If you leave out some questions, try doing every second or every third question. Always do some questions from the end of each exercise because the questions usually get harder at the end. You might use the skipped questions for review before a test.
    - If you are working on a difficult skill or concept, do half the exercise one day and finish the exercise the next day. That is a much better way to learn.
- **Self-tests** at the end of most topics have an "Aim" at the top. If you do not meet the aim, talk to your instructor, find what is causing the trouble, and do some more review before you go on.

- A **Review and Extra Practice** section is at the end of each unit. If there is an area of the unit that you need extra practice in, you can use this. Or, if you want, you can use the section for more review.
- A **Practice Test** is available for each unit. You may:
    - Write the practice test after you have studied the unit as a practice for the end-of-chapter test, OR

    - You might want to write it before you start the unit to find what you already know and which areas you need to work on.

- **Unit Tests** are written after each unit. Again, you must reach the Aim before you begin the next unit. If you do not reach the aim, the instructor will assist you in finding and practising the difficult areas. When you are ready, you can write a B test to show that you have mastered the skills.
- A **Final Test** is to be written when you have finished the book. This final test will assess your skills from the whole book. You have mastered the skills in each unit and then kept using many of them throughout the course. The test reviews all those skills.

Grades Record

You have also been given a sheet to write down your grades. After each test, you can write in the mark. This way you can keep track of your grades as you go through the course. This is a good idea to use in all your courses.

Grade Record – Book 2

Unit	Practice Test	Date of Test A	Test A	Date of Test B	Test B
Example	✔	*September 4, 2020*	*25/33*	*September 7, 2020*	*25/33*
1					
2					
3					
4					
5					
Final Test					

How to Deal with Math Anxiety

Emotions and Learning

Emotions, or what we feel about something, play a big part in how we learn. If we are calm, we learn well. If we are afraid or stressed, we do not learn as well.

Many people are afraid of math. They fear making a mistake. “Math anxiety” is the fear of math. People who suffer from math anxiety may get headaches, sick stomachs, cold hands, or they may just sweat a lot or just feel scared. Math anxiety can happen for a few different reasons:

- Feeling anxious when writing tests
- Negative experiences in a past math class
- Embarrassment in a past math class
- Social pressures and expectations to not like math or not do well in math
- The want to get everything right
- Negative self-message (“I don’t know how to do it,” or “I hate math”)

Math anxiety is a learned habit. If it is learned, it can be unlearned. Most math anxiety comes from bad memories while learning math. It may be from doing badly on a test or asking a question then being made fun of. These bad memories can make learning math hard.

Everyone can learn math. There is no special talent for math. There are some people who are better at math than others, but even these people had to learn to be good at math.

Do You Suffer from Math Anxiety?

Read the list below and put a check mark beside the ones you feel when thinking about or doing math.

- Are your palms moist?
- Is your stomach fluttering?
- Do you feel like you can’t think clearly?
- Do you feel like you would rather do anything else than learn math?
- Are you breathing faster than normal?
- Is your heart pounding?
- Do you feel cold?
- Do you feel sweaty?

If you answered yes to two or more of these items, you may have math anxiety.

If you have math anxiety, a first step to understanding it is to look at where it all started.

Make a list of your experiences with learning math. Think back to the first math experiences you had and write about them. Think about learning math in school from the younger grades to the higher grades and write about your experiences and feelings. Include this class and how you are feeling right now about learning math.

Beside each experience, write if it was a positive or negative experience.

Look at the examples below to give you an idea:

Positive or negative?	**Math experience**
Negative	My teacher in elementary school lined the whole class up in a row and made us play a multiplication game. I could see which question was mine, and I didn't know the answer so I had to figure it out on my fingers before my turn came up. I got the answer right, but I was so nervous that I would be teased because I didn't know the answer off the top of my head. I still don't know my times tables.
Positive	In high school, I could use a calculator to figure out the simple multiplication problems, and then I could figure out the tougher problems without worrying about knowing my times tables.
Negative	Now that I am upgrading my math, I feel nervous every time I even think about opening the book. I want to get all the answers right, and I know that I won't be able to. I really need everything to be right so that I know that I am getting it.

Once you have made a list of experiences, go over the stories with your instructor, or by yourself and try to find some common themes.

- Can you see when you felt anxiety?
- Can you see why you are now anxious about math?
- Is there any experience you could use now to help you feel calmer about math?

Hopefully by examining the beginnings of the anxiety, you can feel more in control of it.

How to Deal with Math Anxiety

Anyone can feel anxiety that will slow down learning. The key to learning is to be the "boss" of your anxiety. Here are an overview of some strategies that may help deal with your anxiety:

- Use breathing exercises
- Think positive math messages
- Know your textbook

- Understand test-taking anxiety

Remember, learning to deal with your math anxiety may take some time. It took you a long time to learn math anxiety, so it will take some time to overcome it.

Use Breathing Exercises

One way to be the “boss” is to relax. Try this breathing exercise.

Breathing Exercise

Start by breathing slowly to the count of four. It may help to close your eyes and count.

Now hold your breath for four counts and then let your breath out slowly to the count of four.

The counting is silent and should follow this pattern: “Breath in, two, three, four. Hold, two, three, four. Breath out, two, three, four. Wait, two, three, four.”

With practice, the number of counts can be increased. This is an easy and good way to relax.

Now, try this exercise quietly and repeat it five times slowly.

Each time you feel anxious about learning, use the breathing exercise to help calm yourself. Ask yourself if what you tried worked. Do you feel calmer?

Think Positive Math Messages

Another way to be the “boss” is to give yourself positive math messages.

Read and think about the positive math messages listed below. Do you say any of those things to yourself?

- If the answer is yes, then great, keep doing that.
- If your answer is no, try to add this little mental trick to your day. The result will probably be that you start to see math as something you can do and that you may even like!

I like math.

I am good at math.

I understand math.

I can relax when I am studying math.

I am capable of learning math.

Math is my friend.

My math improves every day.

I am relaxed, calm and confident when I study math.

I understand math when I give myself a chance.

Math is creative.

Pick three statements that you like and say them to yourself as much as you can in each day. You can also write the statements out on paper and post them around your house so that you read them throughout the day.

Know Your Textbook

Look at the Table of Contents in the front of your textbook. It tells you what you will be learning. You may see some things that you already know, some things that you may have forgotten, and some things that are new to you.

Flip the pages. You can see that the textbook is split into units. Each unit is something to learn.

Each unit has exercises to do. Notice the answers are at the end of the exercise. You can check your answers as soon as you are done. You can also check your answer before moving on if are not sure if you are doing the question right.

At the end of each unit is a self-test. It is a chance for you to see how well you have learned the skills in the unit. If you do well, you can move on. If you don't do well, you can go back and practice those skills.

Knowing your textbook gives you a good skill. If you get frustrated, you can use the Table of Contents to go back and find some help.

Understand Test-Taking Anxiety

There are four reasons people are anxious when writing tests. Any of the four reasons listed below might be the reason a person might feel anxious in a test-taking situation.

1. Not feeling prepared for the test
2. Not sure how to write the test in the best way
3. Feeling too much mental pressure
4. Poor health habits before writing a test

Here is an explanation of each reason and how to work your way out of the anxiety you may feel during tests.

1. Not feeling prepared for the test

Many students feel anxiety about taking math tests because they do not feel prepared for the test. To feel prepared, a student needs to have studied the work and know that they can do the problems they will be given. Get help from your classmates, friends, or your instructor to find out how you can improve your study habits.

Getting ready for a test starts on the first day of class. Everything you do in class and at home is part of that getting ready.

- **Always do as many exercises as you need to help you understand.** Once you understand, do ten more questions, then you will know for sure that you really understand.
- **Always correct your exercises.** It is good to know that you are understanding and getting the questions right. It is also good to know if you are not understanding and need some help.
- **Always do the self-tests.** The self-tests can show things that you are not sure of.
- **Always do the review.** Review is part of this book. It is a chance to go over all the things you have learned in a unit before moving on. It prepares you for what will be on the test.
- **Always do a practice test.** A practice test gives you a chance to see how many questions and what kind of questions are on the test.

2. Not sure how to write the test in the best way

Here are some strategies students should know about how to write a test to do the best as possible on it:

- Before the Test

    1. **Arrive early.** Get out all the supplies you need to do the test (pencils, ruler, calculator, watch, etc.).
    2. **Be comfortable, but alert.** Choose a good spot in the room, and make sure you have enough space to work. Maintain a comfortable posture in your seat, but don't "slouch."
    3. **Stay relaxed and confident.** Keep a good attitude. If you find yourself anxious, take several slow, deep breaths to relax. Don't talk about the test to other students just before entering the room: their anxiety can be contagious.

- During the test.

    1. **Look over the test**. Take a look at the whole test before starting. This takes very little time. Use a highlighter to highlight the questions that you know you can do easily, note key terms, mark the test with comments that come to mind. As you work, put a star beside any questions that you would like to go over again when you finish the test.
    2. **Relax.** Before starting the test, imagine yourself somewhere where you are calm and confident. Go there in your mind. Focus on how good you feel and how in control you are. If you become anxious during the test, in your mind go to the

calming place. Focus on how calm you feel. Then go back to your test.

3. **Read the directions carefully.** This may be obvious, but it will help you avoid careless errors.
4. **Answer questions in a strategic order.**
    - Answer the easy questions first. This will help to build confidence and score points. It may also help you make connections with more difficult questions.
    - Then answer the difficult questions. Work on these harder questions with all the energy of the easier ones.
5. **Review your answers.** Resist the urge to leave as soon as you are done writing. Spend as much time as you can going over your test to see if you:
    - Answered all the questions.
    - Wrote the answers in right.
    - Did not make simple mistakes.

3. Feeling too much mental pressure

There are many reasons why a student may feel mental pressure when writing a test. Listed below are a few main reasons:

- Negative beliefs about one's math abilities
- Low self-esteem when it comes to math
- Too high expectations of success
- Fear that failure or low grades will affect the future
- Feelings of pressure of not wanting to let down family members

When students feel this kind of pressure, it is very hard to feel calm and relaxed about a test. The key to success in a math test is to keep the anxiety at a manageable level. You can do this in two ways:

1. **Change negative self-talk.** Any time a negative thought creeps into your head, it will make it harder to stay positive and relaxed about your test. If you have a negative thought like "I can't do it", try to replace it with a positive thought like "I can do this".
2. **Use relaxing and calming techniques.** Use the calming breathing mentioned earlier in this section. This will help you keep calm. Also, do not study in the last half hour before the test. You will be calmer by spending time relaxing and breathing deeply in that last half hour.

4. Poor health habits before writing a test

When your body and mind are healthy, you will have a better chance of doing well on a test. Eat well, drink plenty of water and get daily exercise. The better you feel, the better you can perform (and a test is a performance!).

Unit 1: Ratio, Rate, & Proportion

Topics

Unit 1 will introduce ratios, rates and proportions.

- Using ratios, we can compare quantities and see the relationship between the numbers in a simple way.
- Rates are used when a ratio is comparing two different kinds of measure.
- A proportion is a statement with two equal ratios. They are be very useful in solving percent problems.

Topic A: Writing Ratios

Introduction to Ratios

Ratio is a comparison of one number or quantity with another number or quantity. Ratio shows the relationship between the quantities.

Ratio is pronounced “rā’shō” or it can be pronounced “rā’shēō.” Check out this YouTube video to listen to someone pronouce the word: How to Pronounce Ratio.

You often use ratios, look at these examples:

- **Scores in games** are ratios. For example, the Penguins won 4 to 3 or the Canucks lost 1 to 5.
- **Directions for mixing** can be ratios. For example, use 1 egg to each cup of milk or mix 25 parts gas to 1 part oil for the motorcycle.
- **Betting odds** are given as ratios. For example, Black Jade is a 3 to 1 favourite or the heavyweight contender is only given a 2 to 5 chance to win.
- **The scale at the bottom of maps** is a ratio. For example, 1 centimetre represents 10 kilometres.
- **Prices** are often given as ratios. For example, 100 grams for $0.79 or 2 cans for $1.85.

For ratios to have meaning you must know what is being compared and the units that are being used. Read these examples of ratios and the units that are used. A general ratio may say “parts” for the units.

A. It rained four days and was sunny for three days last week. The ratio of rainy days to sunny days was $4:3$. ($4:3$ is properly read “4 is compared to 3” but is often read “4 to 3”).

B. The class has 12 men and 15 women registered. The ratio of men to women in the class is $12:15$.

C. At the barbeque, 36 hot dogs and 18 hamburgers were eaten. The ratio of hot dogs eaten to hamburgers eaten is $36:18$.

D. The class spends 3 hours on English and 2 hours on math each day. The ratio of time spent on English compared to math is $3:2$.

Exercise 1

Write the ratios asked for in these questions using the : symbol (for example, $4:1$). Write the units and what is being compared beside the ratio.

A. Powdered milk is mixed using 1 part of milk powder to 3 parts of water. Write a ratio to compare the milk powder to the water.
Answer: $1:3$ ***— 1 part of milk powder to 3 parts of water***

B. One kilogram of ground beef will make enough hamburger for 5 people. Write a ratio to express the amount of ground beef for hamburgers to the number of people.

C. Seventy-five vehicles were checked by the police. Fifteen vehicles did not meet the safety standards, but 60 of them did. Write a ratio comparing the unsafe vehicles to the safe vehicles.

D. The recipe says to roast a turkey according to its weight. For every kilogram, allow 40 minutes of cooking. Write a ratio comparing time to weight.

E. The 4-litre pail of semi-transparent oil stain should cover 24 square metres of the house siding if the wood is smooth. Write the ratio comparing quantity of stain to the smooth wood surface area.

F. The same 4 L of stain will only cover 16 square metres of the house siding if the wood is rough. Write that ratio.

Answers to Exercise 1

B. $1:5$ 1 kg of beef to 5 people

C. $15:60$ 15 unsafe vehicles to 60 safe vehicles

D. $40:1$ 40 minutes to 1 kg of turkey

E. $4:24$ 4 L of stain to 24 m^2 of smooth wood

F. $4:16$ 4 L of stain to 16 m^2 of rough wood

The numbers that you have been using to write the ratios are called the terms of the ratio.

The order that you use to write the terms is very important. Read a ratio from left to right and the order must match what the numbers mean. For example, 3 scoops of coffee to 12 cups of water must be written $3:12$ as a ratio because you are comparing the quantity of coffee to the amount of water.

If you wish to talk about the amount of water compared to the coffee you have, you would say, “Use 12 cups of water for every 3 scoops of coffee” and the ratio would be written $12:3$.

Ratios can be written 3 different ways:

1. Using the : symbol — $2:5$
2. As a common fraction — $\frac{2}{5}$

- The first number in the ratio is the numerator; the second number is the denominator.
- Ratios written as a common fraction are read as a ratio, not as a fraction. Say “2 to 5,” not “two-fifths.”

3. Using the word “to” — 2 to 5

Exercise 2

Use the ratios you wrote in Exercise 1 to complete the chart.

	:	Common fraction	to
A	$1:3$	$\frac{1}{3}$	1 to 3
B			
C			
D			
E			

Answers to Exercise 2

B. $1:5$ or $\frac{1}{5}$ or 1 to 5

C. $15 : 60$ or $\frac{15}{60}$ or 15 to 60

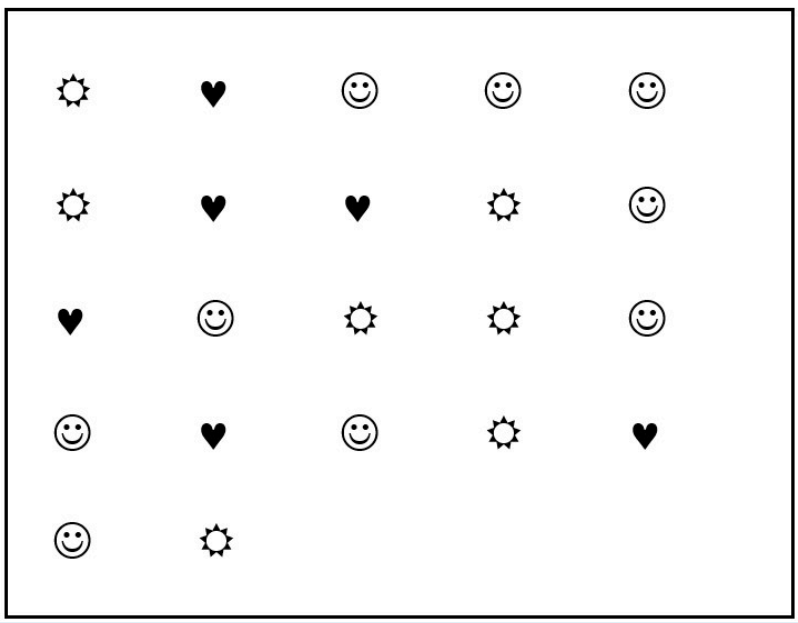
D. $40 : 1$ or $\frac{40}{1}$ or 40 to 1

E. $4 : 24$ or $\frac{4}{24}$ or 4 to 24

F. $4 : 16$ or $\frac{4}{16}$ or 4 to 16

Exercise 3

Use the diagram to write a ratio comparing the quantity of each shape, as asked.



A. ☺ to ♥

B. ☼ to ☺

C. ♥ to ☺

Answers to Exercise 3

A. $9 : 6$ or $\frac{9}{6}$ or 9 to 6

B. $7 : 9$ or $\frac{7}{9}$ or 7 to 9

C. $6 : 9$ or $\frac{6}{9}$ or 6 to 9

Equivalent Ratios

Like equivalent fractions, equivalent ratios are equal in value to each other.

$$10 : 100 = 1 : 10$$

Ratios can be written as common fractions. It is convenient to work with ratios in the common fraction form.

You can then easily:

- Find equivalent ratios in higher terms
- Find equivalent ratios in lower terms
- Find a missing term

Example A

Express $4 : 5$ in higher terms.

$$4 : 5 = \frac{4}{5} \longrightarrow \frac{4}{5} \times \left(\frac{2}{2}\right) \longrightarrow \left(\frac{4 \times 2}{5 \times 2}\right) \longrightarrow \frac{8}{10}$$

$4 : 5$ is equivalent to $8 : 10$

Example B

Express $3:6$ in lower terms.

$$3:6=\frac{3}{6}\longrightarrow\frac{3}{6}\div\left(\frac{3}{3}\right)\longrightarrow\left(\frac{3\div3}{6\div3}\right)\longrightarrow\frac{1}{2}$$

$3:6$ is equivalent to $1:2$

To find equivalent ratios in higher terms, multiply each term of the ratio by the same number. To find equivalent ratios in lower terms, divide each term of the ratio by the same number.

Exercise 4

Write equivalent ratios in any higher term. You may want to write the ratio as a common fraction first. Ask your instructor to mark this exercise.

A. $5:6=\frac{5}{6}\times\left(\frac{3}{3}\right)=\left(\frac{5\times3}{6\times3}\right)=\frac{15}{18}=15:18$

B. $4:3$

C. $10:2$

D. $50:1$

E. $9:4$

F. $3:5$

Answers to Exercise 4
See your instructor.

Exercise 5

Write these ratios in lowest terms—that is, simplify the ratios.

A. $4 : 12 = \frac{4}{12} \div \left(\frac{4}{4}\right) = \left(\frac{4 \div 4}{12 \div 4}\right) = \frac{1}{3} = 1 : 3$

B. $10 : 5$

C. $7 : 21$

D. $20 : 5$

E. $6 : 14$

F. $2 : 4$

G. $6 : 3$

H. $16 : 8$

Answers to Exercise 5

Ratios written as a common fraction or using the word “to” will also be correct in this exercise. The terms must be the same.

A. $1 : 3$

B. $2 : 1$

C. $1 : 3$

D. $4 : 1$

E. $3 : 7$

F. $1 : 2$

G. $2 : 1$

H. $2 : 1$

Exercise 6

Using a colon, write a ratio in lowest terms for the information given.

A. In the class of 25 students, only 5 are smokers. Write the ratio of smokers to non-smokers in the class. (**Note**—you must first calculate the number of non-smokers.)

B. The police issued 12 roadside suspensions to drivers out of the 144 who were checked in the road

block last Friday. Write the ratio of suspended drivers to the number checked.

C. Twenty-seven students registered for the course and 24 completed it. Write a ratio showing number of completions compared to number enrolled.

D. During an hour (60 minutes) of television viewing last night there were 14 minutes of commercials, so there were only 46 minutes of the actual program! Write the ratio of commercial time to program time.

E. For each pair of coins, write the ratio comparing the value. (Use cents.)

i. A nickel to a dime $5 : 10 = 1 : 2$

ii. A nickel to a quarter

iii. A nickel to a dollar

iv. A dime to a nickel

v. A dime to a quarter

vi. A dime to a dollar

vii. A dollar to a dime

Answers to Exercise 6

A. $1 : 4$

B. $1 : 12$

C. $8 : 9$

D. $7 : 23$

i. $1 : 2$

ii. $1 : 5$

iii. $1 : 20$

iv. $2 : 1$

v. $2 : 5$

vi. $1 : 10$

vii. $10 : 1$

Topic A: Self-Test

Mark /12 Aim 10/12

A. Write the definitions.
(3 marks)

i. Ratio

ii. Terms of the ratio

iii. Equivalent ratios

B. Write the ratios asked for in **lowest terms**. Use the colon style like this: $2:1$. Then write the ratio as it is read, like this: two to one.
(4 marks)

i. The campground had three vacant campsites and 47 occupied sites. Write the ratio of occupied sites to vacant sites. Ratio: _____Read: _____ _____

ii. For every ten dogs in the city, only 2 have current dog licences. Write the ratio of licensed dogs to unlicensed dogs. (Find the number of unlicensed dogs first). Ratio: _____Read: _____

C. Simplify these ratios.
(5 marks)

i. $9:12$

ii. $6:4$

iii. $500:1000$

iv. $2:9$

v. $35:15$

Answers to Topic A Self-Test

A. Write the definitions.

i. A **ratio** is a comparison of one number or quantity with another number or quantity. Ratios show the relationship between the quantities or amounts.

ii. **Terms of a ratio** are the numbers used in the ratio, the parts of the ratio.

iii. **Equivalent ratios** are ratios of equal value to each other.

B. Write ratios in lowest terms.

i. $47:3$
Read: ““47 occupied sites to 3 vacant sites.”

ii. $1:4$
Read: “1 licensed dog to 4 unlicensed dogs.”

C. Simplify the ratios.

i. $3:4$

ii. $3:2$

iii. $1:2$

iv. $2:9$

v. $7:3$

Topic B: Rates

When a ratio is used to compare two different kinds of measure (e.g. apples and oranges, or meters and hours), it is called a rate. The denominator must be 1.

Example A

A car can drive 725 km on 55 L of gas. What is the rate in km per L? The ratio of this is $\frac{725\text{ km}}{55\text{ L}}$. Find the rate by making the denominator 1.

Divide $\frac{725}{55} \div \left(\frac{55}{55}\right) = \frac{725 \div 55}{55 \div 55} = \frac{13.18}{1} = 13.18$

The rate is **13.18 km/L**.

Example B

Sue bought 10 lb of oranges for $4.99. What is the rate in cents per pound? The ratio is $\frac{\$4.99}{10\text{ lb}} = \frac{499\text{ cents}}{10\text{ lb}}$. Find the rate by making the denominator 1.

Divide $\frac{499}{10} \div \left(\frac{10}{10}\right) = \frac{499 \div 10}{10 \div 10} = \frac{49.9}{1} = 49.9$

The rate is **49.9 ¢/lb**.

When talking about rate, use the word ‘per’.

In example A, say: “The fuel economy of the car is 13.18 kilometres per litre”.
In example B, say: “The oranges cost 49.9 cents per pound”.

Example C

It takes 60 ounces of grass seed to plant 30 m^2 of lawn. What is the rate in ounces per square metre (m^2)? The ratio is $\frac{60\text{ oz}}{30\text{ m}^2}$. Find the rate by making the denominator 1.

Divide $\frac{60}{30} \div \left(\frac{30}{30}\right) = \frac{60 \div 30}{30 \div 30} = \frac{2}{1} = 2$

The rate is **2 oz/m^2**, or **2 ounces per square metre**.

Exercise 1

Write the following ratios as rates, comparing distance to time.

A. 120 km, 3 hours
B. 27 km, 9 hours
C. 203 km, 29 seconds
D. 444 km, 48 seconds

Answers to Exercise 1

A. 40 km/hour
B. 3 km/hour
C. 7 km/second
D. 9.25 km/second

Exercise 2

Write the following ratios as rates.

A. A leaky faucet can lose 52 litres of water in a week. What is the rate of litres lost per day? (round to two decimal places)
B. A ratio of distance travelled to time is called speed. What is the rate (speed) in kilometres per

hour (km/h)?

i. 45 km, 3 hours
ii. 129 km, 1.5 hours
iii. 65 km, 13 hours

C. Vancouver Island has a population of 734,860, and a land mass of 32,134 square kilometres. What is the rate of number of people per square kilometre? (This is called population density.) Round your answer to the nearest whole number.

D. At rest, the heart beat of a mouse is 30,000 beats per 60 minutes. What is the rate of beats per minute?

Answers to Exercise 2

A. 7.43 L/day

B.
i. 15 km/hour
ii. 86 km/hour
iii. 5 km/hour

C. 23 people/km^2

D. 500 beats/minute

Topic B: Self-Test

Mark /7 Aim 6/7

A. Write the definition.
(1 mark)

i. Rate

B. Write the following ratios as rates. Round people to the nearest person.
(6 marks)

i. 12 cups water, 3 cups sugar
ii. 72 metres, 24 seconds
iii. 1,365,000 people, 4,000 km^2
iv. 5,000 cars on the road, 250 bikes on the road
v. 12 cups of flour, 12 tsp. of baking powder
vi. 8 litres of gas, 2 litres of oil

Answers to Topic B Self-Test

A. A rate is used when a ratio compares two different kinds of measure, and when the denominator is 1.

B.
i. 4 cups of water/cups of sugar
ii. 3 m/second
iii. 341 people km^2
iv. 20 cars/bike
v. 1 cup flour/tsp baking powder
vi. 4 litres gas/litre oil

Topic C: Proportion

A proportion is a statement that two ratios are equal or equivalent. Here are some proportions:

Proportion	Fraction Form	Read like this…
$1 : 2 = 2 : 4$	$\frac{1}{2} = \frac{2}{4}$	1 is to 2 as 2 is to 4
$1 : 4 = 25 : 100$	$\frac{1}{4} = \frac{25}{100}$	1 is to 4 as 25 is to 100
$18 : 9 = 10 : 5$	$\frac{18}{9} = \frac{10}{5}$	18 is to 9 as 10 is to 5
$15 : 20 = 3 : 4$	$\frac{15}{20} = \frac{3}{4}$	15 is to 20 as 3 is to 4

Proportions can be used to solve many math problems. You will soon learn to use proportions to solve problems involving percent. The techniques you practice in the next few pages are important for that problem solving work.

Problems often give incomplete information; that is, one of the terms is missing. To solve such problems, you first find the comparison or ratio that is given. It may be:

- A quantity of one thing that is mixed with a larger quantity of something else
- A scale of measurement given on a map such as 1 cm on the map represents 100 km distance on land
- Cost for a certain number of items

- Time to travel a certain distance

The problem will then give one term of the second ratio in the proportion. For example, if you have been told that 3 heads of lettuce cost $1.49, you may be asked to find the cost of 7 heads of lettuce.

The missing term is the second cost. The proportion will be:

$$\frac{\text{number of heads of lettuce}}{\text{cost}} = \frac{\text{number of heads of lettuce}}{\text{cost}}$$

$$\frac{3}{\$1.49} = \frac{7}{?}$$

$$3 : \$1.49 = 7 :?$$

The most important thing to remember is to keep the order of comparison the same in the first and second ratios in a proportion. If the first ratio compares time to distance then the second ratio in the proportion must compare time to distance.

$$\frac{time}{distance} = \frac{time}{distance}$$

Or it could be:

$$\frac{distance}{time} = \frac{distance}{time}$$

Once you have decided on the order of comparison it is a simple matter to write the proportion using the numbers given in the problem. Use a letter to stand for the missing term.

How would you find a missing term?

- You can use your skills with equivalent ratios (finding higher and lower terms)
- You can use your fraction skills of cross multiplying and then dividing to find the missing term

Using Equivalent Ratios to Solve Proportions

To Solve a Proportion Problem Using Equivalent Ratios

Step 1
Decide on the order of comparison and write a ratio that describes the information given in the

problem. Write a proportion using words of the items that are being compared in fraction form.

Step 2
Write two more ratios with the numbers matching the words in the first ratio. The missing term (number) can be given a letter (ex. *N*).

Step 3
Mentally set the ratio with words (the first ratio) aside.

Step 4
Multiply or divide the complete ratio to find the missing term.

Example A

Use 1 teaspoon of baking powder for every 2 cups of flour. If a recipe uses 6 cups of flour, how much baking powder is needed?The missing term is the teaspoons of baking powder for 6 cups of flour. Call this term *N*.

Step 1
Ratio is $\frac{\text{baking powder}}{\text{flour}}$

Step 2
$$\frac{\text{baking powder}}{\text{flour}} = \frac{1}{2} = \frac{N}{6}$$

Step 3
$$\frac{1}{2} = \frac{N}{6}$$

Step 4
$$\frac{1}{2} \times \left(\frac{3}{3}\right) = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}, \text{ so } \frac{1}{2} = \frac{3}{6}, \text{ so } N = 3$$

Use **3 teaspoons of baking powder for 6 cups of flour.**

Example B

Reports suggest that 3 out of 10 people will at some time miss work due to back pain. If a company has 1,000

employees, how many can be expected to miss work due to back pain.The missing term is the number of people out of 1000 who will miss work due to back pain. Call this term *P*.

Step 1

Ratio is $\frac{\text{people who will miss work}}{\text{all people at work}}$

Step 2

$$\frac{\text{people who will miss work}}{\text{all the people at work}} = \frac{3}{10} = \frac{P}{1000}$$

Step 3

$$\frac{3}{10} = \frac{P}{1000}$$

Step 4

$$\frac{3}{10} \times \left(\frac{100}{100}\right) = \frac{3 \times 100}{10 \times 100} = \frac{300}{1000}, \text{ so } \frac{3}{10} = \frac{300}{1000}, \text{ so } P = 300$$

300 people out of 1,000 people may miss work due to back pain.

Exercise 1

Write the ratio of the words to describe the information given.

A. Three cups of flour to one teaspoon of yeast.
   Answer: $\frac{flour}{yeast}$

B. Four parts oil, ten parts gasoline

C. One centimetre represents 100 kilometres

D. 100 grams for $6.89

E. 3 eggs for each cup of milk

F. 5 men and 7 women

Answers for Exercise 1

B. $\frac{\text{oil}}{\text{gasoline}}$

C. $\frac{\text{centimetres}}{\text{kilometres}}$

D. $\dfrac{\text{grams}}{\text{dollars}}$

E. $\dfrac{\text{eggs}}{\text{milk}}$

F. $\dfrac{\text{men}}{\text{women}}$

Exercise 2

Use equivalent ratios to find the answers.

A. One cup of sugar and four cups of water will make great hummingbird food. How much sugar do you need for 8 cups of water?

$$\frac{sugar}{water} \longrightarrow \frac{1}{4} = \frac{N}{8} \longrightarrow \frac{1}{4} \times \left(\frac{2}{2}\right) = \frac{1 \times 2}{4 \times 2} = \frac{2}{8} \longrightarrow N = 2$$

B. Reports show that for every 100 vehicles checked by police, 20 vehicles do not meet the safety standard. If only 50 vehicles are checked, how many would not meet the safety standard?

C. Four litres of paint covers 24 square metres of wall. How much paint is needed to cover 72 square metres?

D. Powdered milk uses 1 part milk powder to 3 parts water. How much powder should be added to 9 parts water?

Answers for Exercise 2

B. 10 cars would not meet the safety standards

C. 12 litres of paint

D. 3 parts milk powder

Exercise 3

Use equivalent ratios to find the missing term in these proportions.

A. $3 : 5 = Y : 15$

B. $1 : 2 = P : 8$

C. $5 : 7 = 10 : N$

D. $2 : 3 = 8 : \mathrm{W}$

E. $4 : 7 = 16 : A$

F. $1 : 3 = 2 : N$

G. The KX 250 motorcycle uses a mixture of one part oil to 30 parts of gasoline. How much oil must be added to 3,000 mL of gasoline?

Answers for Exercise 3

A. $Y = 9$

B. $P = 4$

C. $N = 14$

D. $W = 12$

E. $A = 28$

F. $N = 6$

G. $N = 100\mathrm{mL}$

Using Cross-Multiplication to Solve a Proportion

Review cross products:

Multiply the numerator of each fraction with the denominator of the other fraction.

$\frac{2}{5} \nearrow\!\!\!\!\searrow \frac{4}{10}$

$$2 \times 10 = 5 \times 4$$
$$20 = 20$$

$2 \times 10 = 20$ and $5 \times 4 = 20$

Therefore: $\frac{2}{5} = \frac{4}{10}$

Remember that when the cross products are the same, the fractions are equivalent.

When finding the missing terms in a proportion, cross-multiplication can be used. Follow the examples carefully.

Example A

$$\frac{2}{3} = \frac{N}{45}$$

Cross multiply:

$$2 \times 45 = 3 \times N$$
$$90 = 3N$$

The idea is to have the unknown term *N* by itself on one side of the equal sign. To do that, remember these things that you already know:

- Division and multiplication are opposite operations
- Whatever is done to one side of an equation or proportion must be done to the other side to keep the equation equal

3*N* means *N* is multiplied by 3. **To get rid of the 3, divide by 3**.

You must also divide the other side of the equation by 3.

$$\frac{90}{3} = \frac{3N}{3}$$

Solve by reducing the $\frac{3}{3}$ and dividing 90 by 3.

$$\frac{90}{3} = \frac{3N}{3}$$

$$\frac{90}{3} = \frac{1N}{1}$$

$$\frac{90}{3} = N$$

$$90 \div 3 = N$$

$$30 = N$$

Reducing the fraction $\frac{3N}{3}$ to $\frac{1N}{1}$ to N is also called *cancelling*. In math, a fraction can be cancelled when the numerator and denominator are the same number.

e.g. $\frac{6P}{6} = \frac{1P}{1} = P$

Example B:

$$\frac{6}{7} = \frac{24}{N}$$

Cross multiply:

$$6 \times N = 7 \times 24$$

$$6N = 168$$

Divide both sides by 6. The 6's with the *N* will cancel (reduce), and the *N* will be alone.

$$\frac{6N}{6} = \frac{168}{6}$$
$$\frac{\cancel{6}\,N}{\cancel{6}} = \frac{168}{6}$$

$$N = 168 \div 6$$

$$N = 28$$

$$\frac{6}{7} = \frac{24}{28}$$

Check by cross-multiplying:

$$\text{Is } 6 \times 28 = 7 \times 24?$$
$$6 \times 28 = 168$$
$$7 \times 24 = 168$$
$$\text{the cross-product } 168 = \text{the cross-product } 168$$
$$\text{Yes - } 6 : 7 = 24 : 28$$

Example C

$$\frac{8}{10} = \frac{N}{80}$$

Cross multiply:

$$8 \times 80 = 10 \times N$$
$$640 = 10N$$

Divide both sides by 10 so *N* will be alone.

$$\frac{640}{10} = \frac{10N}{10}$$
$$\frac{64\,\cancel{0}}{\cancel{10}} = \frac{\cancel{10}\,N}{\cancel{10}}$$
$$64 = N$$

To Solve a Proportion Problem Using Cross-Multiplication

Step 1
Write the first ratio using the information given.

Step 2
Write the proportion, using a letter in place of the missing term. Be sure the **order of comparison is the same** in both the first and second ratios in your proportion.

Step 3
Write the proportion in the fraction form. (Try to **simplify** the ratio **before** you do all the calculations).

Step 4
Cross-multiply and set the cross-products equal to each other.

Step 5
Divide both sides of the equation by the number with the unknown term.

Step 6
Check by putting your answer back into the original proportion and cross-multiplying.

Exercise 4

Practise using cross-multiplying to find the missing term in these proportions.

A.

$$\frac{5}{8} = \frac{N}{32}$$

$$5 \times 32 = 8 \times N$$

$$160 = 8N$$

$$\frac{160}{8} = \frac{8N}{8}$$

$$160 \div 8 = N$$

$$20 = N$$

B. $\frac{4}{N} = \frac{24}{30}$

C. $\frac{12}{4} = \frac{18}{x}$

D. $\frac{y}{6} = \frac{20}{12}$

E. $4 : 15 = 8 : N$

F. $W : 100 = 6 : 50$

Answers to Exercise 4

B. $N = 5$

C. $x = 6$

D. $y = 10$

E. $N = 30$

F. $W = 12$

The numbers in a ratio often are common fractions, decimals or mixed numbers. Follow exactly the same steps that you have been using to solve whole number proportions. The calculations will use your skills with fractions.

Example A

$2\frac{1}{4} : 3 = N : 7$

Rewrite the proportion:

$\frac{2\frac{1}{4}}{3} = \frac{N}{7}$

Cross-multiply:

$$2\frac{1}{4} \times 7 = 3 \times N$$

$$\frac{9}{4} \times \frac{7}{1} = 3 \times N$$

$$\frac{63}{4} = 3N$$

$$\frac{63}{4} \div \frac{3}{1} = \frac{3 \times N}{3} \longrightarrow \frac{63}{4} \times \frac{1}{3} \qquad = N$$

$$\frac{63}{12} = N \qquad \longrightarrow 5\frac{3}{12} \longrightarrow 5\frac{1}{4} = N$$

Exercise 5

Practise using cross-multiplying to find the missing term in these proportions.

$$6.5 : 5 = 13 : A$$

$$\frac{6.5}{5} = \frac{13}{A}$$

A. $6.5A = 65$

$$A = 65 \div 6.5$$

$$A = 10$$

B. $3\frac{1}{2} : 2 = N : 8$

C. $9 : 6 = 4\frac{1}{2} : N$

D. $7.5 : B = 10 : 20$

E. $3.75 : 5 = 9x$

F. $4\frac{1}{8} : A = 3 : 6$

Answers to Exercise 5

B. $N = 14$

C. $N = 3$

D. $B = 15$

E. $x = 12$

F. $A = 8\frac{1}{4}$ or 8.25

Exercise 6

A. Joanne can walk 18 km in 3 hours. How far can she walk, at the same rate in 5½ hours?

B. The taxes on the property valued at $300,000 are valued at $5,000. At the same rate of taxation, what would the taxes be on the smaller lot down the street which is valued at $240,000?

C. One B.C. road map has a scale of 0.5 centimetres equal to 10 kilometres. Complete the chart by calculating actual driving distances in kilometres between some B.C. places.The proportions will be $\frac{0.5}{10} = \frac{\text{cm given in chart}}{\text{actual distance in km}}$

Places in B.C.	Number of cm between places on the map	Actual distance in kilometres
Kelowna and Vernon	2.5 cm	
Burns Lake and Vanderhoof	5.5 cm	
TaTa Creek and Skookumchuk	0.75 cm	
Kitimat and Terrace	3.3 cm	

D. The directions on the lawn fertilizer say to spread 1.7 kg over 100 m^2 of lawn.

   i. How much fertilizer is needed for a 130 m^2 lawn?

   ii. How much fertilizer for a 75 m^2 lawn?

Answers to Exercise 6

A. 33 km

B. $4,000

C.

Places in B.C.	Number of cm between places on the map	Actual distance in kilometres
Kelowna and Vernon	2.5 cm	50 km
Burns Lake and Vanderhoof	5.5 cm	110 km
TaTa Creek and Skookumchuk	0.75 cm	15 km
Kitimat and Terrace	3.3 cm	66 km

D.

i. 2.21 kg

ii. 1.275 kg

Topic C: Self-Test

Mark /20 Aim 17/20

A. Solve these proportions.
(6 marks)

i. $N : 14 = 28 : 56$

ii. $3 : 11 = N : 22$

iii. $50 : 45 = 10 : N$

iv. $4\frac{1}{5} : Y = 3 : 2$

B. **(14 marks)**

i. Get a map of BC, a map of Canada, and a map of your city or town.

ii. Find the scale on each map (usually at the bottom) and write down the ratio of map distance to the actual distance.

iii. With another student or an instructor, calculate actual distances between places by measuring the distance on the map and working out the proportion according to the scale given. Do at least three distance calculations on each map.

Ask your instructor to mark your work.

Answers to Topic C Self-Test

A.

i. $N = 7$

ii. $N = 6$

iii. $N = 9$

iv. $Y = 2\frac{4}{5}$ or 2.8

B. See your instructor.

Unit 1 Review: Ratio, Rate, & Proportion

Questions

1. Write the ratios asked for in the questions. Reduce when needed.
    a. It snowed for two days and was sunny for five days last week. Write a ratio to compare snowy days to sunny days.
    b. The class spends 6 hours a week on math and 8 hours on English. Write the ratio of hours spent on math to English.
    c. One kilogram of pork will make enough sausages for 7 people. Write a ratio to express the amount of pork for sausages to the number of people.
    d. A jam recipe calls for 8 cups of blue berries to 3 cups of sugar. Write a ratio to express the amount of berries to sugar.

2. Write the ratios in lowest terms.
    a. $14 : 21$
    b. $8 : 24$
    c. $15 : 150$
    d. $9 : 54$

3. Write the following ratios as rates.
    a. 16 cups of water to 4 cups of sugar
    b. 150 kilometres to 1.25 hours
    c. 465,000 people to 3,000 square kilometres
    d. $3.99 to 10 pounds of apples

4. Use cross multiplication to solve the proportions.
    a. $\frac{5}{8} = \frac{P}{24}$
    b. $\frac{S}{100} = \frac{8}{25}$

c. $\frac{4}{N} = \frac{8}{40}$

d. $\frac{4}{14} = \frac{F}{68}$

5. Use cross multiplication to solve the proportions.

a. $\frac{2\frac{1}{4}}{5} = \frac{A}{30}$

b. $\frac{11}{8} = \frac{4\frac{1}{8}}{P}$

c. $\frac{3\frac{3}{4}}{5} = \frac{8}{X}$

d. $\frac{2\frac{2}{3}}{3} = \frac{A}{16}$

Answers to Unit 1 Review

1. a. $2 : 5$
   b. $6 : 8$
   c. 1 kg : 7 people
   d. 8 cups berries : 3 cups sugar
2. a. $2 : 3$
   b. $1 : 3$
   c. $1 : 10$
   d. $1 : 6$
3. a. 4 cups water per cup of sugar
   b. 120 km/hour
   c. 155 people per km^2

d. $0.40 per pound of apples

4. a. 15
   b. 32
   c. 20
   d. 19.43

5. a. $13\frac{1}{2}$
   b. 3
   c. $10\frac{2}{3}$
   d. $14\frac{2}{9}$

TEST TIME!

Ask your instructor for the Practice Test for this unit.

Once you've done the Practice Test, you need to do the Unit 1 test.

Again, ask your instructor for this.

Good luck!

Unit 2: Percent

Percents are another form of fractions and are used in many everyday situations. Interest rates, credit card charges, taxes, pay deductions, increases and decreases are all calculated with percent. Percents are a convenient way to express part of the whole thing because the unwritten denominator is always 100.

- the denominator of 100 is not written or said.
- a percent sign % follows the number and is read as “percent”.
- the whole thing is 100%
- $100\% = 1$

Remember that *cent* is the Latin word meaning *a hundred* or a *hundredth*. And *per* is a Latin word meaning *by* or *for each*. So, *percent* means *for each hundred*.

Topic A: Introducing Percent

Reading and Writing Percents

To write a percent:

- Write the number in the usual way
- Place the percent sign after the numerals
    - 50%
    - $5\frac{1}{2}\%$ or 5.5%
    - $\frac{3}{4}\%$ or 0.75%

To read a percent:

- Read the numbers in the usual way
- Say “percent” after the number
    - 16%: say “sixteen percent”
    - $4\frac{1}{2}\%$: say “four and one-half percent”
    - 0.25%: say “twenty-five hundredths percent,” or “one-quarter percent,” or “point two five percent”

Exercise 1

Write these percents using numerals and a percent sign. Note that the mixed numbers may be expressed with common fractions or decimals.

A. Thirty-four percent 34%

B. Twelve percent

C. Four-fifths percent

D. One hundred sixteen and three-tenths percent
E. Thirteen percent
F. Six and one-fifth percent
G. Ninety-four and one-half percent

Answers to Exercise 1

B. 12%
C. 0.8% or $\frac{4}{5}\%$
D. 116.3% or $116\frac{3}{10}\%$
E. 13%
F. 6.2% or $6\frac{1}{5}\%$
G. 94.5% or $94\frac{1}{2}\%$

Exercise 2

A. 62% sixty-two percent
B. $37\frac{1}{2}\%$
C. 202%
D. $\frac{3}{4}\%$
E. 18.3%
F. $14\frac{1}{2}\%$
G. $100\frac{1}{2}\%$

Answers to Exercise 2

B. Thirty-seven and one-half percent

C. Two hundred two percent
D. Three-quarters percent
E. Eighteen and three-tenths percent
F. Five-tenths percent or one-half percent or zero point five percent
G. One hundred percent

Changing Decimals to Percents

Writing equivalent fractions is an important math skill.

Equivalent common fractions, decimals, and percents all represent the same amount.

Equivalent fractions, decimals, and percentages

Fractions	Decimals	Percentages
$\frac{1}{2}$	0.5	50%
$\frac{3}{10}$	0.3	30%

You need the skill of writing equivalent fractions for working with percents.

To change any number to a percent, multiply the number by 100% and place a percent sign % after the product.

Remember this shortcut for multiplying by 100?

$$4.27 \times 100 = 427$$
$$0.287 \times 100 = 28.7$$
$$53 \times 100 = 5300$$

The shortcut is: **When multiplying by 100, move the decimal point two places to the right.**

Example A

Change these numbers to a percent.

$$\begin{array}{llcl} 1 & 1 \times 100\% & = & 100\% \\ 0.25 & 0.25 \times 100\% & = & 25\% \\ 0.8 & 0.8 \times 100\% & = & 80\% \\ 0.375 & 0.375 \times 100\% & = & 37.5\% \end{array}$$

So…

To change a decimal to a percent, move the decimal point two places to the right and then write the percent sign after the number.

Example B

Change each decimal to a percent.

$$0.125 = \overset{\curvearrowright}{0.125} = 12.5\%$$

$$1.375 = \overset{\curvearrowright}{1.375} = 137.5\%$$

If the decimal point moves to the end of the number it is not necessary to write the decimal point. Remember that zeros at the beginning of a number are also not necessary.

$$0.24 = \overset{\curvearrowright}{0.24} = \cancel{0}.24\% = 24\%$$

$$0.05 = \overset{\curvearrowright}{0.05} = \cancel{0.0}\,5\% = 5\%$$

If the decimal is a tenth (one decimal place), it will be necessary to add a zero. If you are changing a whole number to a percent, add two zeros.

$$0.4 = \overset{\curvearrowright}{0.40} = 40\%$$

$$1.7 = \overset{\curvearrowright}{1.70} = 170\%$$

$$2 = \overset{\curvearrowright}{2.00} = 200\%$$

Exercise 3

Change these decimals to percents.

	Decimal	× 100% Move decimal 2 places to right	= Percent
A.	0.75	$\overset{\curvearrowright}{0.75}$	$= 75\%$
B.	0.33		
C.	0.1		
D.	0.0025		
E.	0.9		
F.	0.325		
G.	0.0625		
H.	3		

Answers to Exercise 3

B. 33%

C. 10%
D. 0.25%
E. 90%
F. 32.5%
G. 6.25%
H. 300%

Changing Percents to Decimals

Review dividing by 100:

$$47.39 \div 100 = 0.4739$$
$$429 \div 100 = 4.29$$
$$3.824 \div 100 = 0.03824$$

To divide by 100, move the decimal point two places to the left.

Example A

Change each percent to a decimal or mixed number.

$$\begin{aligned} 58\% &= 58 \div 100 = .58 = 0.58 \\ 20\% &= 20 \div 100 = .2 = 0.2 \\ 6\% &= 6 \div 100 = .06 = 0.06 \\ 110\% &= 110 \div 100 = 1.10 \end{aligned}$$

So…

To change a percent to a decimal, divide by 100 (move the decimal point two places to the **left**) and remove the percent sign.

Example B

Change each percent to a decimal.

$$\begin{aligned} 75\% &= 75.0\% &&= 0.75 \\ 12\% &= 12.0\% &&= 0.12 \\ 37.5\% &= 37.5\% &&= 0.375 \\ 125\% &= 125.0\% &&= 1.25 \\ 5\% &= 5.0\% &&= 0.05 \\ 4.6\% &= 4.6\% &&= 0.046 \end{aligned}$$

Some notes to remember:

- If there is no decimal point in the percent, place the decimal point **after** the last numeral and **then** divide by 100. $24\% = 24.0\% = 0.24$
- It may be necessary to **prefix** zeros. (This means adding zeros in front of the number, if needed). $6\% = 6.0\% = 0.06$
- A zero at the right of a decimal is not needed and may be left off.
  $40\% = 40.0\% = 0.40 = 0.4$

Exercise 4

Change each percent to its decimal equivalent.

	Percent	÷ 100% Move decimal 2 places to left	= Decimal
A.	23%	$\overset{\curvearrowleft}{23.}$	$= 0.23$
B.	1%		
C.	112%		
D.	10.3%		
E.	36%		
F.	147%		

Answers to Exercise 4

B. 0.01
C. 1.12
D. 0.103
E. 0.36
F. 1.47

To change a percent containing a common fraction to a decimal, do this:

- Change the common fraction in the percent to a decimal in the percent.
- Divide by 100 (move the decimal 2 places to the **left**).

Example C

$$\begin{aligned}
3\tfrac{1}{2}\% &= 3.5\% & 3.5\% \div 100 &= .035 = 0.035\\
37\tfrac{1}{2}\% &= 37.5\% & 37.5\% \div 100 &= .375 = 0.375\\
\tfrac{1}{4}\% &= 0.25\% & 0.25\% \div 100 &= .0025\\
17\tfrac{1}{3}\% &= 17.\overline{3}\% & 17.\overline{3}\% \div 100 &= 0.17\overline{3}
\end{aligned}$$

Exercise 5

Change each percent to its decimal equivalent.

A. $8\frac{4}{5}\% =$ $8.8\% = 0.088$

B. $4\frac{1}{2}\% =$

C. $56\frac{3}{4}\% =$

D. $1\frac{3}{5}\% =$

E. $112\frac{1}{2}\% =$

F. $2\frac{3}{8}\% =$

G. $5\frac{1}{4}\% =$

Answers to Exercise 5

B. 0.045

C. 0.5675

D. 0.016

E. 1.125

F. 0.02375

G. 0.0525

Changing Common Fractions to Percents

To change any number to a percent, multiply the number by 100% and place the percent sign % after

the product.

There are two methods you can use to change a common fraction to a percent.

Method One:

To change a common fraction to an equivalent percent, **multiply the common fraction by 100%.**

Example A

$\frac{3}{4} = \_\_\_\_\_\%$

	$\frac{3}{4}$
Multiply by 100%	$\frac{3}{4} \times 100\%$
Convert 100% to a fraction.	$\frac{3}{4} \times \frac{100}{1}\%$
Simplify the fractions by dividing the numerator and denominator by 4.	$\frac{3}{\cancel{4}\,1} \times \frac{\cancel{100}\,25}{1}\%$
The 4 cancels and 100 is reduced to 25.	$\frac{3}{1} \times \frac{25}{1}\%$
Multiply 3 by 25%.	$3 \times 25\%$
	75%

$1\frac{1}{5} = \_\_\_\_\_\%$

	$1\frac{1}{5}$
Convert number to a fraction.	$\frac{6}{5}$
Multiply by 100%.	$\frac{6}{5} \times 100\%$
Convert 100% to a fraction.	$\frac{6}{5} \times \frac{100}{1}\%$
Simplify the fractions by dividing the denominator and numerator by 5.	$\frac{6}{\cancel{5}1} \times \frac{\cancel{100}\,20}{1}\%$
The 5 cancels and 100 is reduced to 20.	$\frac{6}{1} \times \frac{20}{1}\%$
Multiply 6 by 20%.	$6 \times 20\%$
	120%

Exercise 6

Multiply by 100% to change each common fraction to an equivalent percent.

A. $\frac{4}{5}$ $\frac{4}{5} \times 100\% = 80\%$

B. $\frac{1}{5}\% =$

C. $\frac{9}{10}\% =$

D. $1\frac{1}{2}\% =$

E. $\frac{7}{10}\% =$

F. $3\frac{3}{4}\% =$

G. $\frac{1}{2}\% =$

Answers to Exercise 6

B. 20%

C. 90%

D. 150%

E. 70%

F. 375%

G. 50%

Method Two:

To change a common fraction to an equivalent percent, first write the common fraction as a decimal. Then multiply the decimal by 100% (move the decimal point two places to the **right**).

Example A

1. $\frac{3}{8} = \_\_\_\_\_\%$
2. Use long division to write the fraction as a decimal.
3. $\frac{3}{8} = 0.375$
4. Move the decimal two spots to the right.

$$0.375 = 37.5$$

Convert $\frac{3}{8}$ to a percentage

	$\frac{3}{8} = \_\_\_\_\_\%$
Use long division to write fraction as a decimal.	$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ 2\,4\downarrow\downarrow \\ 60\downarrow \\ \underline{56}\downarrow \\ 40 \end{array}$
	$\frac{3}{8} = 0.375$
Move the decimal point two places to the right and add the percent sign.	$0.375 = 37.5\%$

Convert $\frac{1}{3}$ to a percentage

	$\frac{1}{3} = \_\_\_\_\_\%$
Use long division to write fraction as a decimal.	$\begin{array}{r} 0.33\overline{3} \\ 3\overline{)1.000} \end{array}$
	$\frac{1}{3} = 0.33\overline{3}$
Move the decimal point two places to the right and add the percent sign.	$0.33\overline{3} = 33.\overline{3}\%$ also written as $33\frac{1}{3}\%$

Convert $\frac{11}{12}$ to a percentage

	$\frac{11}{12} = \_\_\_\_\_\%$
Use long division to write fraction as a decimal.	$0.916\overline{6}$ $12\overline{)11.000}$ $10\ 8\downarrow\downarrow$ $20\downarrow$ $12\downarrow$ 80 $\underline{72}$ 8
	$\frac{11}{12} = 0.91\overline{6}$
Move the decimal point two places to the right and add the percent sign.	$0.916\overline{6} = 91.\overline{6}\%$

Exercise 7

A. $\frac{1}{12} = 0.08\overline{3} \qquad 0.08\overline{3} \times 100\% = 8.\overline{3}\%$

B. $\frac{1}{8}$

C. $\frac{5}{8}$

D. $\frac{7}{8}$

E. $\frac{2}{3}$

F. $\frac{5}{16}$

G. $\frac{5}{6}$

H. $\frac{4}{9}$

Answers to Exercise 7

B. 12.5%
C. 62.5%
D. 87.5%
E. $66.\overline{6}\%$
F. 31.25%
G. $83.\overline{3}\%$
H. $44.\overline{4}\%$

The method you use to change a common fraction to a percent will depend on the numbers you are working with. Choose whichever method seems easier for the situation. You will also memorize many equivalencies as you work with them. But you should definitely memorize:

- $\frac{1}{3} = 33\frac{1}{3}\%$
- $\frac{2}{3} = 66\frac{2}{3}\%$

Changing Percents to Common Fractions

You know that percents are a form of fraction with an unwritten denominator of 100. A % sign is used.

To change any percent to a decimal, common fraction, or mixed number, divide by 100 and remove the percent sign.

To change a percent to a common fraction:

- Write the numerals in the percent as the numerator.
- Write 100 as the denominator. (Remember that the line in a fraction can be a divided by sign, so $58\% = \frac{58}{100}$ is the same as $58 \div 100$.)
- Remove the % sign.
- Simplify the fraction: $\frac{58}{100} = \frac{29}{50}$

Example A

Write each percent as a common fraction.

$$38\% = \frac{38}{100} \div \left(\frac{2}{2}\right) = \frac{38 \div 2}{100 \div 2} = \frac{19}{50}$$

$$25\% = \frac{25}{100} \div \left(\frac{25}{25}\right) = \frac{25 \div 25}{100 \div 25} = \frac{1}{4}$$

$$3\% = \frac{3}{100}$$

Note that **percents greater than or equal to 100 become improper fractions** which will be rewritten as mixed numbers.

$$110\% = \frac{110}{100} = 1\frac{10}{100} = 1\frac{1}{10}$$

$$\begin{array}{r} 1 \\ 100\overline{)110} \\ \underline{100} \\ 10 \end{array}$$

$$120\% = \frac{120}{100} = 1\frac{20}{100} = 1\frac{1}{5}$$

$$\begin{array}{r} 1 \\ 100\overline{)120} \\ \underline{100} \\ 20 \end{array}$$

Remember 100% is the whole thing. $100\% = 1$.

Exercise 8

Change each percent to a common fraction. Simplify to lowest terms.

A. $31\% =$
B. $11\% =$
C. $2\% =$
D. $20\% =$

E. $75\% =$

F. $100\% =$

G. $750\% =$

Answers to Exercise 8

A. $\frac{31}{100}$

B. $\frac{11}{100}$

C. $\frac{1}{50}$

D. $\frac{1}{5}$

E. $\frac{3}{4}$

F. 1

G. $7\frac{1}{2}$

Percents Less Than 1%

Sometimes a percent **smaller than 1%** is used. For example, you will hear amounts such as $\frac{1}{4}\%$ or $\frac{1}{8}\%$ or $\frac{1}{2}\%$ on the news about the Bank of Canada rate and the rise and fall of inflation. These are small amounts. Sometimes the expression "$\frac{1}{2}$ of a percentage point" is used instead of "$\frac{1}{2}\%$".

What is $\frac{1}{4}\%$?

$\frac{1}{4}\%$ is $\frac{1}{4}$ of 1%

$1\% = \frac{1}{100}$, so $\frac{1}{4}$ of $1\% = \frac{1}{4} \times \frac{1}{100} = \frac{1}{400}$

$\frac{1}{4}\% = 0.25\% = 0.0025$

What is $\frac{1}{2}\%$?

$\frac{1}{2}\% = \frac{1}{2} \text{ of } 1\% = \frac{1}{2} \times \frac{1}{100} = \frac{1}{200}$

$\frac{1}{2}\% = 0.5\% = 0.005$

To work with percents less than 1%, change the percent to a decimal by dividing by 100 (move decimal point two places to the left).

$0.2\% = 0.002$
$0.75\% = 0.0075$

If the percent is expressed as a common fraction, do this:

- Write the common fraction percent as a decimal percent.
- Divide by 100 (move decimal point two places left).

\begin{equation}\begin{split} \tfrac{1}{2}\% &= 0.5\% &= 0.005 \\ \tfrac{1}{4}\% &= 0.0025\% &= 0.0025 \\ \tfrac{1}{8}\% &= 0.00125\% \ & = 0.00125 \end{split}\end{equation}

Exercise 9

Change each percent to an equivalent decimal.

A. $\frac{1}{2}\% =$
B. $0.6\% =$
C. $\frac{3}{10}\% =$
D. $\frac{3}{5}\% =$
E. $0.75\% =$
F. $\frac{3}{4}\% =$
G. $0.5\% =$
H. $\frac{1}{4}\% =$
I. $0.125\% =$
J. $\frac{5}{8}\% =$

Answers to Exercise 9

A. 0.005
B. 0.006
C. 0.003
D. 0.006
E. 0.0075
F. 0.0075
G. 0.005
H. 0.0025
I. 0.00125
J. 0.00625

16⅔%, 33⅓%, 66⅔%, 83⅓% ...

These percents will become repeating decimals. For example:

$$33\tfrac{1}{3}\% = 33.\overline{3}\% = 0.33\overline{3}$$

$$66\tfrac{2}{3}\% = 66.\overline{6}\% = 0.66\overline{6}$$

It is usually more convenient to use the common fraction equivalent of these percents. Memorize them, or make a note on a special paper and post it near your work space.

$$33\tfrac{1}{3}\% = 33\tfrac{1}{3} \div 100 = \tfrac{100}{3} \times \tfrac{1}{100} = \tfrac{1}{3}$$

$$66\tfrac{2}{3}\% = 66\tfrac{2}{3} \div 100 = \tfrac{200}{3} \times \tfrac{1}{100} = \tfrac{2}{3}$$

$16\frac{2}{3}\% = \frac{1}{6}$

$33\frac{1}{3}\% = \frac{1}{3}$

$66\frac{2}{3}\% = \frac{2}{3}$

$83\frac{1}{3}\% = \frac{5}{6}$

Review of Equivalent Common Fractions, Decimals, and Percents

Complete this chart. These are equivalents that you will often use, so use this chart for reference. Memorize as many equivalents as you can. You may wish to put other equivalents on the chart.

Common Fraction	Decimal	Percent
$\frac{1}{4}$		
	0.5	
		75%
$\frac{1}{8}$		
	0.375	
		62.5%

Answers to Review of Equivalent Common Fractions, Decimals, and Percents

Common Fraction	Decimal	Percent
$\frac{1}{4}$	0.25	25%
$\frac{1}{2}$	0.5	50%
$\frac{3}{4}$	0.75	75%
$\frac{1}{8}$	0.125	12.5%
$\frac{3}{8}$	0.375	37.5%
$\frac{5}{8}$	0.625	62.5%

Topic A: Self-Test

Mark /15 Aim 13/15

A. Write these percents in numeral form.
(2 marks)

i. Sixty-two and one-half percent

ii. One hundred six and one-fifth percent

B. Write these percents in words.
(2 marks)

i. 72%

ii. $\frac{3}{4}\%$

C. Change the percents to decimal fractions.
(5 marks)

i. 32%

ii. 18.5%

iii. 125%

iv. $\frac{4}{5}\%$

v. $\frac{2}{3}\%$

D. Change these percents to common fractions in lowest terms.
(4 marks)

i. 16%

ii. 20%

iii. 106%

iv. 75%

E. Change these common fractions to equivalent percents.
(2 marks)

i. $\frac{4}{5}$

ii. $\frac{3}{8}$

Answers to Topic A Self-Test

i. 62.5% or $62\frac{1}{2}\%$

ii. 106.2% or $106\frac{1}{5}\%$

i. Seventy-two percent

ii. Three-quarters percent

i. 0.32

ii. 0.185

iii. 1.25

iv. 0.008

v. $0.00\overline{6}$

i. $\frac{4}{25}$

ii. $\frac{1}{5}$

iii. $1\frac{3}{50}$

iv. $\frac{3}{4}$

i. 80%

ii. 37.5%

Unit 2 Review: Percent

Questions

1. Change these decimals to percents.
    a. 0.75
    b. 0.156
    c. 0.03
    d. 0.0035
    e. 0.625
    f. 0.048
    g. 3.45

2. Change each percent to a decimal.
    a. 59%
    b. 39.5%
    c. 1%
    d. $3\frac{1}{2}\%$
    e. $1\frac{4}{5}\%$
    f. $4\frac{3}{5}\%$

3. Change each fraction to an equivalent percent.
    a. $1\frac{3}{4}$
    b. $\frac{1}{5}$
    c. $4\frac{3}{20}$
    d. $\frac{9}{20}$
    e. $2\frac{1}{3}$

4. Change each percent to a common fraction. Simplify your answer.
    a. 16%
    b. 25%
    c. 110%
    d. 95%
    e. 650%
    f. 284%
    g. $\frac{1}{4}\%$
    h. $\frac{1}{8}\%$
    i. 0.125%
    j. $\frac{5}{8}\%$
    k. $\frac{3}{4}\%$

5. Change each percent to a common fraction. These few should be memorized.
    a. $33\frac{1}{3}\%$
    b. $66\frac{2}{3}\%$
    c. $16\frac{2}{3}\%$
    d. $83\frac{1}{3}\%$

Answers to Unit 2 Review

1.
    a. 75%
    b. 15.6%
    c. 3%

d. 0.35%
e. 62.5%
f. 4.8%
g. 345%

2. a. 0.59
b. 0.395
c. 0.01
d. 0.035
e. 0.018
f. 0.046

3. a. 175%
b. 20%
c. 415%
d. 45%
e. 233.3%

4. a. $\frac{4}{25}$
b. $\frac{1}{4}$
c. $1\frac{1}{10}$
d. $\frac{19}{20}$
e. $6\frac{1}{2}$
f. $2\frac{21}{25}$
g. $\frac{1}{400}$
h. $\frac{1}{800}$
i. $\frac{0.125}{100}$ or $\frac{1}{800}$
j. $\frac{0.7}{100}$ or $\frac{7}{1000}$

k. $\frac{3}{400}$

5.

a. $\frac{1}{3}$

b. $\frac{2}{3}$

c. $\frac{1}{6}$

d. $\frac{5}{6}$

TEST TIME!

Ask your instructor for the Practice Test for this unit.

Once you've done the Practice Test, you need to do the Unit 2 test.

Again, ask your instructor for this.

Good luck!

Unit 3: Working with Percent

Topics

In this unit you will learn to solve three types of percent problems:

- Finding a given percent of a number (finding the part).
- Finding what percent one number is of another number (finding the %).
- Finding a number when a percent of it is given (finding the whole).

Each type of percent problem can be solved using the following proportion:

$$\frac{\text{is (part)}}{\text{of (whole)}} = \frac{\%}{100}$$

Both ratios in this proportion use the same order of comparison because in the ratio $\frac{\%}{100}$, the % represents a part and 100 is the whole. That is, the % is a part of the whole.

Percent problems involve knowing three pieces of information:

1. the part (the “is” part)
2. the whole (the “of” part)
3. the percent

You will be given two pieces of information and you will find the third. That is, the problems will give two terms of the proportion, and you will solve for the missing term. Because these are problems of percent, the 100 is always known to you and will always be in the same position in the proportion.

Remember how to use cross multiplication to solve a proportion:

$$\frac{\mathrm{N}}{4} = \frac{6}{8} \longrightarrow 4 \times 6 = 8 \times \mathrm{N} \longrightarrow 24 = 8\mathrm{N} \longrightarrow \frac{24}{8} = \frac{8N}{8} \longrightarrow \frac{24}{8} = \frac{\cancel{8}\,\mathrm{N}}{\cancel{8}} \longrightarrow 3 = \mathrm{N}$$

Topic A: Finding a Percent of a Number

$$\frac{\text{is}(\text{part})}{\text{of}(\text{whole})} = \frac{\%}{100}$$

In problems in which you find a percent of a number, the **missing term** is the **part**. You will be given the % which is always 100.

Example A

What is 25% of 40?

- Part (is) = *N*
- Whole (of) = 40
- % = 25

$$\frac{is}{of} = \frac{\%}{100} \longrightarrow \frac{N}{40} = \frac{25}{100}$$

Solve the proportion.

Simplify if possible:

$$\frac{N}{40} = \frac{25}{100}$$

$$\frac{N}{40} = \frac{\cancel{25}\ 1}{\cancel{100}\ 4}$$

$$\frac{N}{40} = \frac{1}{4}$$

Cross multiply:

$$N \times 4 = 40 \times 1$$

$$4N = 40$$

Divide:

$$N = 40 \div 4 = 10$$

25% of $40 = 10$

Example B

What is 20% of 18?

- Part (is) = *N*
- Whole (of) = 18
- % = 20

$$\frac{is}{of} = \frac{\%}{100} \longrightarrow \frac{N}{18} = \frac{20}{100}$$

Solve the proportion:

$$\frac{N}{18} = \frac{20}{100}$$

$$\frac{N}{18} = \frac{\cancel{20}\,1}{\cancel{100}\,5}$$

$$\frac{N}{18} = \frac{1}{5}$$

$$5N = 18$$

$$N = 18 \div 5 = 3\tfrac{3}{5}$$

20% of $18 = 3\frac{3}{5}$

The following examples all ask you to find a percent of a number. The missing term is the **part (the "is" part)**. Look at the examples carefully so you'll recognize the wording.

- What is 14% of 60? $\frac{is}{of} = \frac{\%}{100} \longrightarrow \frac{N}{60} = \frac{14}{100}$
- Find 10% of 27. $\frac{is}{of} = \frac{\%}{100} \longrightarrow \frac{P}{27} = \frac{10}{100}$
- 5% of 15 is _____. $\frac{\%}{100} = \frac{is}{of} \longrightarrow \frac{5}{100} = \frac{X}{15}$

- 75% of $12 =$ _____. $\dfrac{\%}{100} = \dfrac{is}{of} \longrightarrow \dfrac{75}{100} = \dfrac{K}{12}$

In percent problems, the number after the word **of** usually represents the **whole**.

Exercise 1

Solve each problem by setting up the proportion $\dfrac{\text{is (part)}}{\text{of (whole)}} = \dfrac{\%}{100}$.

A. 20% of $18 =$
B. 19% of $200 =$
C. 25% of $44 =$
D. 6% of $110 =$
E. 3% of $33 =$
F. 30% of $64 =$
G. 50% of 60 is?
H. 30% of 40 is?
I. What is 72% of $\$425$?
J. What is 20% of 85?

Answers to Exercise 1

A. $3\frac{3}{5}$ or 3.6
B. 38
C. 11
D. $6\frac{3}{5}$ or 6.6
E. $\frac{99}{100}$ or 0.99
F. $19\frac{1}{5}$ or 19.2
G. 30
H. 12
I. $\$306$
J. 17

Percents Greater Than or Equal to 100%

Remember that $100\% = 1$.

100% of anything is the whole thing. If you spend 100% of your pay cheque, you spend the whole thing. If you get 100% on a test, you have the whole thing correct.

If you have **more than 100%**, you have **more than the whole thing**. If you spend 110% of your paycheque, you spent more than you earned, and you may be in trouble! It is hard to get more than 100% on a test unless the instructor has given bonus marks for extra questions. You may hear of percents more than 100% in increases, such as costs of housing or inflation. For example, “The Browns just sold their house and made a 200% profit.” This means they got back what they paid and two times more!

If a percent is less than (<) 100, it is less than the whole thing.

- $120\% \text{ of } 50 = 60$

If a percent is 100, it equals the whole thing.

- $90\% \text{ of } 50 = 45$

If a percent is more than (>) 100, it is more than the whole thing.

- $100\% \text{ of } 50 = 50$

Exercise 2

Look at the percent. Is it 100? Circle the correct answer for each question. Do not solve the problems.

A. $200\% \text{ of } 10$ is:

i. equal to 10
ii. less than 10
iii. greater than 10

B. 50% of 0.25 is:

i. equal to 0.25
ii. less than 0.25
iii. greater than 0.25

C. 90% of 75 is:

i. equal to 75
ii. less than 75
iii. greater than 75

D. $33\frac{1}{3}\%$ of 15 is:

i. equal to 15
ii. less than 15
iii. greater than 15

E. 100% of 100 is:

i. equal to 100
ii. less than 100
iii. greater than 100

F. 127% of 936 is:

i. equal to 936
ii. less than 936
iii. greater than 936

Answers to Exercise 2

A. iii) greater than 10
B. ii) less than 0.25
C. ii) less than 75
D. ii) less than 15
E. i) equal to 100
F. iii) greater than 936

Exercise 3

Review p. 70 first and use the proportion method to solve these questions.

A. $16\frac{2}{3}\%$ of $12 =$
B. What is 60% of 15?
C. 75% of 144 is?
D. 30% of $90 =$
E. What is $37\frac{1}{2}\%$ of 80?
F. 25% of 52 is?
G. Find 8.2% of 300.
H. 260% of 45 is?
I. What is 109% of 200?
J. 98.75% of $50 =$

Answers to Exercise 3

A. 2
B. 9
C. 108
D. 27
E. 30
F. 13
G. 24.6
H. 117
I. 218
J. 49.375

Taxes

The amount of tax to be paid is calculated by finding a percent of a number. The tax rate is usually given as a percent. The basic proportion for these problems is:

$$\frac{\text{tax (part)}}{\text{taxable amount (whole)}} = \frac{\%\text{tax}}{100}$$

Please note that the tax rates used in the questions in this book are for the year 2010 and are subject to change.

The British Columbia Harmonized Sales Tax (HST) is 12%. In B.C., the provincial portion of the harmonized sales tax does not have to be paid on children's clothes, food, books, gasoline and diesel fuel, and other special items.

Example A

How much HST (12%) will be charged on a new kitchen table that cost $125?Use proportion:

$$\frac{HST}{\$125} = \frac{12}{100}$$
$$100HST = 12 \times 125 \qquad = \$1,500$$
$$HST = \$1,500 \div 100 = \$15.00$$

HST on a $125 table is **$15.00**.

Exercise 4

Find the total cost of each item. All are to be taxed with HST.

	Purchase Price	HST 12%	Total Cost
A	Clothes: $130		
B	Washing Machine: $589		
C	New Car: $10,000		
D	Shoes: $59.99		

Answers to Exercise 4

	Purchase Price	HST 12%	Total Cost
A	Clothes: $130	$15.60	$145.60
B	Washing Machine: $589	$70.68	$659.68
C	New Car: $10,000	$1,200	$11,200
D	Shoes: $59.99	$7.20	$67.19

Income tax is charged at different percentages according to the amount of a person's taxable income. The first $28,000 of taxable income is taxed at 17%. Note that other tax rules and charges may apply in real situations.

Example B

If a person's taxable income for the year is $23,400, what amount of income tax will that person pay? To use the proportion method, do this:

$$\frac{\text{tax}}{\text{income}} \longrightarrow \frac{\text{T}}{\$23,400} = \frac{17}{100}$$

The tax is the part. The income is the whole.

Solve for $3,978.

The income tax on $23,400 is **$3,978**.

Exercise 5

Calculate the income tax for the annual taxable earnings listed. These amounts are all under $28,000, so the tax rate is 17%.

A. $18,500
B. $27,620
C. $15,365
D. $25,900

Answers to Exercise 5

A. $3,145
B. $4,695.40
C. $2,612.05
D. $4,403

Cross-Border Shopping

The Canadian (CAN) and American (US) dollars are not equal in value. The **exchange rate** (the value of one Canadian dollar compared to a dollar from another country) changes often; the current rate is usually available from banks, on the news, in the newspapers and on a web site. In the winter of 2010, the Canadian dollar was around $0.92 of an American dollar (ratio is $\$1.00\text{ CAN} : \$0.92\text{ US}$), so CAN money was valued at 92% of US money.

To find the value of one US dollar in Canadian funds, use this proportion:

$$\frac{\$1\text{ CAN}}{\$0.92\text{ US}} = \frac{N\text{ CAN}}{\$1\text{ US}}$$

$N = \$1 \div \$0.92 = \$1.086$, so US money was valued at 109% of CAN money.

Note that the proportion changes as the exchange rate changes.

What if you buy in the United States?

- Change the US cost to the Canadian equivalent (multiply by 109%).
- If you have more than the purchases allowed (call the **Canada Border Service Agency** for information), the Canadian Customs charge **duty** on the Canadian value of your purchases. The percent of the duty (the rate) varies according to what the item is, where it was made and the duty rates of the day. For example, duty on poultry is 12.5%, on non-US cotton 25%, and on liquor 110%!
    - Duty is gradually being eliminated under the Canada-US Free Trade Agreement. If an item is made in North America, there is no duty charged because of NAFTA (North American Free Trade Agreement)
- HST (12%) is charged on the duty and on the Canadian value of the purchases (which includes any US sales taxes).

Look at this example (assume $1.00 CAN = $0.92 US).

Men's leather shoes, US price	$64.80
US sales tax 6%	3.89
US total cost	68.69
Equivalent cost in CAN funds (US cost $\times$ 109%)	
$\frac{N}{68.69} = \frac{109}{100}$	$74.87
Duty on leather shoes is 22.8%	$17.07
HST (12%) on CAN value plus duty	
12% of $74.19 and 16.92 together	$11.03

The total cost of a pair of leather shoes priced at $64.80 in the United States will be the American price in Canadian funds + duty + HST = **$102.97 CAN**.

Exercise 6

For each item, do the calculations using the duty and tax rates given. Assume $1.00 US is $1.09 CAN.

A. Groceries, US price $75.

i. 3% US sales tax
ii. US total cost
iii. Equivalent value in CAN funds
iv. Duty at 10.2% (No HST on food)
v. Total cost in Canadian funds

Answers to Exercise 6

A.
i. $2.25
ii. $77.25
iii. $84.20
iv. $8.59
v. $92.79

Increases and Decreases, Discounts and Mark-ups

Increases (amount changing to more) and decreases (amount changing to less) are often given as a percent. For example,

- The Insurance Corporation of B.C. increased car insurance rates by 3.3% in March 2007.
- The number of acute care beds at the local hospital has decreased by 28% in the last year.
- The new work contract provides a 4% wage increase in the first year and a 2½% increase in the second year.

The amount of an increase or decrease is calculated by **finding a percent of a number**. When the percent of an increase or decrease is given, the proportion is:

$$\frac{\text{amount of increase or decrease}}{\text{whole amount}} = \frac{\text{increase or decrease \%}}{100}$$

Discounts are a form of decrease. The discount is the amount taken off a price; it is the price reduction.

Sale prices (discounted prices) may be advertised as:

- "All items 20% off."
- "Everything in stock reduced by 25% to 50%."
- "33⅓% savings!"
- "2% discount for cash."

Decrease and discount problems may need to be solved in several steps. Sometimes the problems ask for:

- the amount of the decrease (may be called the "saving") (1 step).
- the amount left **after** the decrease (2 steps).

Example A

The sign says, "All winter coats 40% off." How much money will you **save** on a coat originally priced at $128.99?What is 40% of $128.99?

One step problem

$$\frac{\text{decrease}}{\text{original price}} \longrightarrow \frac{\text{D}}{\$128.99} = \frac{40}{100}$$

$$\begin{aligned} 100\text{D} &= 40 \times \$128.99 \\ \text{D} &= \$5,519.60 \div 100 \quad = \$51.596 \\ &\quad \text{round to nearest cent} = \$51.60 \end{aligned}$$

You will save **$51.60**.

Example B

The couch and chair are advertised in a 33⅓% price reduction sale. How much will you **pay** for a couch and chair originally priced at $798?

Two step problem

First: Find the amount of savings (the decrease).

Recall from p. 70 that it's best to substitute ⅓ for 33⅓%.

$$\frac{saving}{full\ cost} \longrightarrow \frac{S}{798} = \frac{1}{3}$$

Second: Subtract the savings from the original amount.

$$\text{original amount} - \text{savings} = \text{sale price}$$

1. Find the amount of savings.

$$\frac{\text{S}}{798} = \frac{1}{3}$$

$$798 \times \frac{1}{3} = \text{S}$$

$$\frac{798}{1} \times \frac{1}{3} = \text{S}$$

$$266 = \text{S}$$

2. Find the sale price.

$$\begin{aligned} \text{original amount} - \text{savings (decrease)} &= \text{sale price} \\ \$798 - \$266 &= \$532 \end{aligned}$$

The couch and chair will cost **$532 on sale** (plus HST of course, but you do not have to calculate tax for this problem).

Exercise 7

Solve these problems. Round all answers to the nearest cent.

A. The employees agreed to take a 5% pay cut (reduction) so no-one will be laid off. If the pay rate was $15.50 per hour, how much less per hour will the workers earn?

B. "All shoes 25% off," says the sign. What will be the sale price of a pair of dress shoes originally priced at $69.98?

C. The work force at the factory has to be reduced by 16⅔% over the next two years. Early retirements, attrition (not replacing people who leave) and some lay-offs will be used. The work force is 3,000 people right now. What is the planned size of the work force in two years?

Answers to Exercise 7

A. $0.78

B. $52.48

C. 2,500 people

Increases and mark-ups are calculated in the same way as decreases and discounts. However, an increase or mark-up is **added** to the original amount.

Example C

The auto insurance rate increased 19%. The basic insurance rate for Don's car was $550 before the increase. What is the basic insurance after the increase?

1. Calculate the amount of the increase. Find 19% of $550.

$$\frac{\text{amount of increase}}{\text{present cost}} \longrightarrow \frac{\text{N}}{550} = \frac{19}{100}$$

$$\begin{aligned} 100\text{N} &= 550 \times 19 \\ \text{N} &= 104.5 \end{aligned}$$

2. **Add** the amount of increase to the original amount.

$$\text{original amount} + \text{increase} = \text{new insurance cost}$$
$$\$550 + \$104.50 = \$654.50$$

Don's new basic insurance is **$654.50**.

Mark-ups are the amount added to the cost price before an item is resold. Many factors must be considered when businesses decide on the percent of the mark-up:

- all costs of operating a business
- the profit wanted
- the community the business is in
- the competition the business has

For example, the mark-up on leather shoes may be 45%, but on running shoes it may be 60%. Kitchen appliances might have a 42% mark-up, while lawn mowers might have a 55% mark-up.

Example D

A shoe seller pays $40.00 per pair of running shoes from the factory. The shoe seller makes the mark up 75%. What is the selling price of the shoes?What is 75% of $40.00?

$$\frac{\text{mark-up cost}}{\text{original cost}} \longrightarrow \frac{\text{N}}{40} = \frac{75}{100}$$

$$\begin{aligned} 100\text{N} &= 3,000 \\ \text{N} &= 30 \end{aligned}$$

75% of $40.00 is **$30.00**.

Add the mark up to the original cost to get the selling price of the shoes:

$\$40 + \$30 = \$70.00$.

Wage Increase

Having a wage increase at work is always a good thing! Often the raise will be given as a percentage. That means that everyone will see more money on their pay cheque, but they will each have a different amount because they all get paid a different amount to start with.

Example E

The boss at A-1 House Painting will give a 1.5% wage increase to the 10 employees.

a. 3 staff are paid the minimum wage of $13.85 an hour.
b. the other 7 staff are paid $21.00 an hour.

$$\frac{\text{increase}}{\text{present wage}} \longrightarrow \frac{\text{I}}{13.85} = \frac{1.5}{100}$$

a. What is 1.5% of \$13.85? The new wage will

$$\begin{aligned} 100\text{I} &= 13.85 \times 1.5 \\ \text{I} &= \$0.21 \end{aligned}$$

be the old wage plus the increase:
$\$13.85 + \$0.21 = \$14.06$ per hour

$$\frac{\text{increase}}{\text{present wage}} \longrightarrow \frac{\text{I}}{21} = \frac{1.5}{100}$$

b. What is 1.5% of \$21.00? The new wage will be the

$$\begin{aligned} 100\text{I} &= 21 \times 1.5 \\ \text{I} &= \$0.32 \end{aligned}$$

old wage plus the increase:
$\$21.00 + \$0.32 = \$21.32$ per hour

Exercise 8

Solve the problems. Round money to the nearest cent.

A. If the mark-up on the craft supplies was set at 75%, calculate the selling price for these items and complete the chart.

Cost Price to Business	Mark-up (75%)	Selling Price
Silk Flowers: \$1.48	$\frac{\text{N}}{1.48} = \frac{75}{100} = \$1.11$	$\$1.48 + \$1.11 = \$2.59$
Stuffing: \$4.50/bag		
Beads: \$3.20/dozen		

B. The population of the town has increased 30% since the pulp mill was built. The population before the pulp mill was 8,436 people. What is the population now?
(Round to the nearest person.)

C. The wage contract gave the workers a 4% increase in the first year and a 2½% increase in the second year. If the hourly rate of pay was \$12.45 before the new contract, calculate the following:

i. The amount of the increase per hour in the first year.

ii. The hourly pay rate in the first year. (old pay + increase = first year rate)

iii. The amount of pay increase in the second year. (Note—use the new hourly pay rate from the first year to calculate the increase for the second year.)

iv. The hourly pay rate in the second year. (first year rate + increase = second year rate)

Answers to Exercise 8

A.

Cost Price to Business	Mark-up (75%)	Selling Price
Silk Flowers: $1.48	$1.11	$2.59
Stuffing: $4.50/bag	$3.38	$7.88
Beads: $3.20/dozen	$2.40	$5.60

B. 10,967 people

C.

i. $0.50

ii. $12.95

iii. $0.32

iv. $13.27

Commission and Tips

Salespeople may receive a **commission** as part or all of their pay. The business owner pays the salesperson an agreed-upon percent of the selling price of the product.

- Real estate agents are paid by commission on their sales.
- Car and truck salespeople may be paid a small salary per month, but their main income is the commission on the vehicles they sell.

Tips are appreciation payments for service. The customer gives tips directly to the worker. Taxi drivers, waiters, bellhops and chambermaids in hotels often receive a minimal hourly wage. A large part of their earnings is from **tips**. In restaurants, expect to leave at least a 15% tip for adequate service.

To calculate the amount of a commission (or a tip), find the percentage of the total amount using the proportion:

$$\frac{commission\ \square}{total\ amount} = \frac{commission\ \%}{100}$$

The commission is the part.
The total amount is the whole.

Commission problems often have several steps. You may have to:

- add together several items to find the total of sales.
- subtract a base amount for which salespeople do not receive a commission.
- add the amount of commission to the basic wage to find out how much the person earned.

Example A

The bill for the excellent dinner at the restaurant was $56.40. The service had been good and the waiter very pleasant so Bill and Diane wanted to leave at least a 15% tip.

i. To calculate the tip, find 15% of $56.40.

$$\frac{\text{tip}}{\text{bill}} \longrightarrow \frac{\text{N}}{56.40} = \frac{15}{100}$$

The tip is the part. The bill is the whole. Bill will round

$$\begin{aligned} 100\text{N} &= 56.40 \times 15 \\ \text{N} &= \$8.46 \end{aligned}$$

this amount to $8.50

ii. How much will he pay for the meal and the tip?

cost of dinner	$56.40
+tip	$8.50
	$64.90

In a **real** situation, we would probably round the amount of the bill to the nearest dollar and then calculate the tip.

Example B

The salespeople at XW Ford receive a monthly salary of $1,000. They also receive a 12% commission on any sales over $35,000 in a month. This means they are expected to sell $35,000 worth of vehicles every month to earn the $1,000 salary. If a saleswoman made $54,000 in sales one month, what would her gross earning be?You are asked to find the gross monthly earnings. What do you know?

- She earns $1,000 per month.
- She earns 12% commission on sales over $35,000.
- She had $54,000 in sales.

1. Find the amount of commissionable sales.Subtract the base amount for which she will not earn a commission from her total sales.
$$\begin{aligned}\text{total sales} - \text{base amount} &= \text{amount of sales that commission will be paid for} \\ \$54,000 - \$35,000 &= \$19,000 \text{ in commissionable sales}\end{aligned}$$
2. Calculate the commission.What is 12% of $19,000?
$$\frac{\text{commission}}{\text{commissionable sales}} \longrightarrow \frac{\text{X}}{19,000} = \frac{12}{100}$$
$$\begin{aligned}100\text{X} &= 19,000 \times 12 \\ 100\text{X} &= 228,000 \\ \text{X} &= \$2,280\end{aligned}$$
3. Add the salary and commission to find gross earnings.
$\$1,000 + \$2,280 = \$3,280.$The saleswoman earned **$3,280**.

Exercise 9

A. A clerk sold $18,000 worth of clothes last year. He was paid a 15% commission. How much was his commission?

B. Mr. Green receives a weekly salary of $325 plus a commission of 10% on all sales he makes over $1,500. Last week Mr. Green sold $3,500 worth of merchandise. How much money did he earn last week?

C. The final bill at the restaurant is $160 and you want to leave a 15% tip.

   i. What amount of tip should you leave?

   ii. What is the total cost (bill and tip)?

Answers to Exercise 9

A. $2,700

B. $525

C.
   i. $24.00
   ii. $184.00

More Problems for Finding a Percent of a Number

A. To successfully pass the course, the student must get at least 80% on the test. The test is out of 125. What mark will give the student 80%?

B. George made a 12½% down payment on a new car which cost $3,200. How much was the down payment?

C. In B.C., employers are required to pay 6% holiday pay to all employees. Holiday pay is added to regular salary if a paid vacation is not taken. The young grocery clerk who earns $8.00 an hour worked 25 hours last week.

   i. What amount of holiday pay is he eligible for?

   ii. His employer pays holiday pay on each cheque to part-time employees. What are his total earnings for the week? (salary + holiday pay)

D. Nutrition experts recommend that no more than 30% of the food calories a person consumes should be from fats. Foods such as fatty meat, dairy products with lots of butter fat, cooking oils, margarine, some salad dressings, and nuts contain a high percentage of fat. If a person's daily intake of calories is 2,560, what is the most number of calories that should be from fat?

Answers to More Problems for Finding a Percent of a Number

A. 100 marks

B. $400

C.
   i. $12.00
   ii. $212.00

D. 768 calories

Topic A: Self-Test

Mark /11 Aim 9/11

A. Answer the following.
(4 marks)

i. 6% of $30 =$

ii. $\frac{1}{4}\%$ of 48 is?

iii. Find 131% of 400.

iv. Write the general proportion that can be used to solve percent problems.

B. Solve the problems. (2 marks each, except c)
(7 marks)

i. The $125,000 house is insured for 85% of its value against fire damage. How much money should the owner receive if the house is destroyed by fire?

ii. Charlotte sells leisure clothes in her small town for a large national company. She receives $500 a month, and a 20% commission on all monthly sales over $1 000. What are her monthly earnings if her total sales are $2,300?

iii. The barbeque was originally priced at $599 but Jack bought it during a 35% off, end-of-season sale.

a. What was the sale price? **(1 mark)**

b. Calculate the harmonized sales tax (12%). **(1 mark)**

c. Give the total cost of Jack's barbeque. **(1 mark)**

Answers to Topic A Self-Test

A.
i. 1.8
ii. 0.12
iii. 524
iv. $\frac{part}{whole} = \frac{\%}{100}$, or $\frac{is}{of} = \frac{\%}{100}$

B.
i. $106,250
ii. $760.00
iii.
a. $389.35
b. $46.72

c. $436.07

Unit 3 Review: Working with Percent

Questions

1. Solve each problem by setting up a proportion.
    a. 150% of 7
    b. 12.5% of 48
    c. 20% of 16
    d. 18% of 40
    e. $\frac{1}{4}\%$ of 20
    f. 0.5% of 122

2. Use the proportion method to solve these questions.
    a. 75% of 12
    b. $3\frac{1}{4}\%$ of 200 is?
    c. What is 16% of 34?
    d. 90% of 75 is
    e. Find 10% of 27
    f. $16\frac{2}{3}\%$ of 163

3. Find the total cost of each item, using 12% HST.

	Item	Cost	HST (12%)	Total Cost
A	T-shirt	$14.99	$\frac{X}{14.99} = \frac{12}{100}$ $X = \$1.80$	$\$1.80 + \$14.99 = \$16.79$
B	40″ flat screen TV	$699.00		
C	Tent	$168.79		
D	Drill and bit set	$248.99		
E	Shoes	$79.98		

4. Solve these percent problems.
    a. All pants are 25% off said the sign in the window. What will be the sale price of a pair of pants originally priced at $59.78?
    b. 30% of the forest will be cut down in the next three years. The forest is 150 acres large. How many acres will be left after the cut?
    c. All winter clothes are on sale for 15% off. A set of gloves cost $18.00. How much will you save?
    d. Rent will increase 10% on Oliver's apartment on January 1. His rent was $460.00 per month. What will his new rent be?
    e. The French Bread Bakery staff will get a one-time 15% pay increase on their Christmas pay check. If Manon works 63 hours at $12.78/hour for that pay check, how much will she get paid? (Figure out the regular pay first and then add on the 15% increase.)
    f. The bill at the restaurant was $38.00. Add a 20% tip. What did the customer pay in total?
    g. The mark up on a pair of fancy running shoes is 45%. The shoes cost the buyer $38.00. How much will they be sold for after mark-up?

Answers to Unit 3 Review

1. a. 10.5
   b. 6
   c. 3.2
   d. 7.2
   e. 0.05
   f. 0.61

2. a. 9
   b. 6.5
   c. 5.44
   d. 67.5
   e. 2.7
   f. 27.16

3.

	Item	Cost	HST (12%)	Total Cost
A	T-shirt	$14.99	$1.80	$16.79
B	40″ flat screen TV	$699.00	$83.88	$782.88
C	Tent	$168.79	$20.25	$189.04
D	Drill and bit set	$248.99	$29.88	$278.87
E	Shoes	$79.98	$9.60	$89.58

4. a. $44.83
   b. 105 acres
   c. $2.70
   d. $506.00
   e. $925.91
   f. $45.60
   g. $55.10

TEST TIME!

Ask your instructor for the Practice Test for this unit.

Once you've done the Practice Test, you need to do the Unit 3 test.

Again, ask your instructor for this.

Good luck!

Unit 4: More Working with Percent

Topic A: Finding What Percent One Number is of Another

$$\frac{\text{is (part)}}{\text{of (whole)}} = \frac{\%}{100}$$

In problems where you must find what percent one number is of another, the **missing term** is the **percent**. You will be told the part (is) and the whole (of), you know the 100, and you solve for the missing percent.

Example A

4 is what percent of 5?

- 4 is the part (is)
- 5 is the whole (of)
- % is unknown (call it P)

Write the proportion:

$$\frac{4}{5} = \frac{P}{100}$$

Cross-multiply to solve:

$$\begin{aligned} 4 \times 100 &= 5 \times P \\ 400 &= 5P \\ 400 \div 5 &= P \\ 80\% &= P \end{aligned}$$

Be sure to write the percent sign – %.

In percent problems, the number after “of” **usually** is the whole.The number close to “is” **usually** is the part. You may find it helpful to think “is over of”.

Like this: $\frac{\text{is}}{\text{of}}$.

An equal to sign (=) can substitute for “is”.

12 is what percent of 15?

$\frac{\text{is}}{\text{of}}$ will help to find $\frac{\text{part}}{\text{whole}}$.

$$\frac{12\ (\text{is})}{15\ (\text{of})} = \frac{N}{100}$$

Example B

What percent of 85 is 60?

- 60 = the part (close to “is”)
- 85 = the whole (after “of”)
- % = the unknown

Set up the proportion: $\frac{\text{is}}{\text{of}}$

$$\frac{60}{85} = \frac{P}{100}$$

Simplify:

$$\frac{\cancel{60}\ 12}{\cancel{85}\ 17} = \frac{P}{100}$$

$$\frac{12}{17} = \frac{P}{100}$$

Cross-multiply to solve:

$12 \times 100 = 1,200$, and $17 \times P = 17P$

$$1,200 = 17P$$
$$1,200 \div 17 = P$$
$$70.588\% = P$$

Round to **70.6%**.

Exercise 1

The following examples ask you to find what percent one number is of another. The **missing term** is the **percent**. Look carefully at the wording and decide which number is the part (close to "is") and which number is the whole thing (after "of").Write the proportion but do not solve the problem.

A. 3 is what % of 6?

B. 12 is _____% of 5?

C. What % of 27 is 9?

D. What % of ½ is ¼?

E. _____% of 50 is 25?

F. _____% of 64 = 48

Answers to Exercise 1

A. $\dfrac{3}{6} = \dfrac{N}{100}$

B. $\dfrac{F}{100} = \dfrac{12}{5}$

C. $\dfrac{9}{27} = \dfrac{X}{100}$

D. $\dfrac{\frac{1}{4}}{\frac{1}{2}} = \dfrac{P}{100}$

E. $\dfrac{X}{100} = \dfrac{25}{50}$

F. $\dfrac{48}{64} = \dfrac{N}{100}$

Exercise 2

Solve each question by first setting up the proportion $\frac{\text{is (part)}}{\text{of (whole)}} = \frac{\%}{100}$. Review p. 70 as necessary.

A. 3 is what percent of 60?
B. 3 is what percent of 4?
C. 1 is what percent of 3?
D. What % of 50 is 35?
E. What percent of 350 is 42?
F. 15 is _____% of 12.
G. 14 is _____% of 700.
H. What percent of 96 is 12?
I. 2 is _____% of 125.
J. 46 is _____% of 40.

Answers to Exercise 2

A. 5%
B. 75%
C. $33.\overline{3}\%$
D. 70%
E. 12%
F. 125%
G. 2%
H. 12.5%
I. 1.6%
J. 115%

Exercise 3

Solve the following by setting up the proportion.

A. 16 is _____% of 64.
B. 17 is _____% of 85.
C. What % of 52 is 13?
D. What percent of 125 is 75?
E. 1 is _____% of 200.
F. 36 = _____% of 12.
G. _____% of 72 = 27.
H. _____% of 48 = 18.
I. 125 = _____% of 75.
J. What % of 18 is 24?

Answers to Exercise 3

A. 25%
B. 20%
C. 25%
D. 60%
E. 0.5%
F. 300%
G. 37.5%
H. 37.5%
I. $166.\overline{6}$ or $166\frac{2}{3}\%$
J. $133.\overline{3}$ or $133\frac{1}{3}\%$

Finding the Percent of an Increase or Decrease

You learned in to find the amount of an increase (gain) or decrease (loss) when given the percent of the increase or decrease.

Now you are going to find the percent of the increase or decrease when you are given the amounts. This is called the rate of the increase or decrease.

Problems which ask you to find the percent of increase or decrease often involve two steps:

1. Find the amount of change (either increase or decrease) by finding the difference between the

two amounts given. Subtract to find the difference.

2. Find the percentage of increase/decrease. Always compare the change (amount of increase or decrease) to the amount before the change (the original amount) using this proportion.

$$\frac{\text{amount of increase or decrease}}{\text{original amount}} = \frac{P}{100}$$

Example A

The rent went from $375 a month to $427.50 a month. What is the percent of the increase?

1. Find the change (the amount of increase) by finding the difference between the amounts.
   $\$427.50 - 375 = \$52.50$ The amount of increase is $52.50.
2. Find the % of increase.
   The amount of increase is $52.50.
   The original amount (the amount before the increase) is $375. What % of $375 is 52.50?

$$\frac{X}{100} = \frac{52.50}{375}$$
$$375X = 5250$$
$$X = \frac{5250}{375}$$
$$X = 14\%$$

The rent increase is **14%**.

Example B

The hours of operation at the college were reduced from 35 hours a week to 30 hours a week. What is the percent of this cut in operations?

1. Find the amount change (a decrease) by finding the difference between the amounts.
   $35 \text{ hours } - 30 \text{ hours } = 5 \text{ hours}$ The amount of decrease is 5 hours.

2. Find the % of increase.
   Decrease is 5 hours.
   Original amount is 35 hours.What percent is 5 of 35?

$$\frac{5}{35} = \frac{P}{100}$$
$$500 = 35P$$
$$\frac{500}{35} = P$$
$$14\frac{2}{7}\% = P$$

The hours of operation at the college were cut **14⅔%**.

Exercise 4

Solve the following problems.

A. Ms. Lister's bi-weekly unemployment cheque increased from $405 to $435. What percent increase is this?

B. Joan weighed 72 kg before she went on a programme of strict exercise and careful eating. She now weighs 60 kg. What is the percent of her weight loss?

C. The car dealership gives a special deal if the customer does not have a trade-in and pays cash. The dealers will only charge $10,650 for a car listed at $12,000. What is the percent savings in this deal?

D. A regular toilet uses 20 litres of water per flush. By purchasing a new low flow toilet, the water use is 6 litres per flush. What is the percent savings of water per flush if the new tank is used?

Answers to Exercise 4

A. 7.4% increase

B. 16.6% decrease

C. 11.25% savings

D. 70% savings

Other Problems

Many situations compare one number to another.

- 24 out of 25 on the test
- 6 out of 10 people are overweight
- The government spends 27¢ of every federal tax dollar on the national debt

These numbers are often more easily thought about if written as a percent.

$$\frac{\text{is (part)}}{\text{of (whole)}} = \frac{\%}{100}$$

The following problems ask you to find what percent one number is of another. Often several steps are involved to calculate the part or to calculate the whole (as in question e) You may be asked to use the % after you find it. Remember the whole thing = 100%.

Exercise 5

Solve the following problems.

A. The Doal family net income is $2,300 per month. Their mortgage payment is $750 each month. What percent is the mortgage payment of their monthly income?

B. Jean played on the college volleyball team and missed a lot of classes when she travelled to tournaments. She missed nine of the 42 English classes last semester.

   i. What percent of her English classes did she miss?

   ii. What percent of her English classes did she attend?

C. Four women and six men serve on the Village Council. What percent of the council members are women?

D. Gail bought a $500 G.I.C. (Guaranteed Investment Certificate) one year ago. She was delighted to receive her annual interest cheque of $52.50 today. What percent interest did Gail's G.I.C. pay for that year?

E. If a math book had 320 pages and you still had 110 pages left to do, what percent of the book had you finished? (2 steps)

Answers to Exercise 5

A. 32.6% of monthly income

B.

   i. 21.4% of English classes

   ii. 78.6% of English classes

C. 40% of members are women
D. 10.5% interest
E. 65.625% or 65⅝% of the book

School Grades

When looking at test results, the mark shows how you did on the test.

If you get $\frac{7}{10}$ on a test, you know you got 7 answers right, and 3 answers wrong.

Sometimes it is also helpful to see your mark as a percentage.

Example A

By solving for N, the percentage can be found:

$$\frac{7}{10} = \frac{N}{100}$$
$$7 \times 100 = N \times 10$$
$$\frac{\cancel{700}\ 70}{\cancel{10}} = \frac{\cancel{10}\ N}{\cancel{10}}$$
$$\frac{70}{1} = \frac{N}{1}$$
$$70 = N$$

So, $\frac{7}{10} = 70\%$.

Now, you can see that the test mark of $\frac{7}{10}$ equals 70%.

Example B

The test result was $\frac{15}{43}$, what was the percent on the test?

$$\begin{aligned}
\frac{15}{43} &= \frac{N}{100} \\
15 \times 100 &= N \times 43 \\
\frac{1500}{43} &= \frac{\cancel{43}\ N}{\cancel{43}} \\
\frac{1500}{43} &= \frac{N}{1} \\
34.88\% &= N
\end{aligned}$$

If you round, $N = 35\%$.

Not such a great mark!

Example C

Find the percent of the following grade: $\frac{89}{97}$.

$$\begin{aligned}
\frac{89}{97} &= \frac{P}{100} \\
89 \times 100 &= P \times 97 \\
\frac{8900}{97} &= \frac{\cancel{97}\ P}{\cancel{97}} \\
\frac{8900}{97} &= \frac{P}{1} \\
91.75\% &= P
\end{aligned}$$

$P = 91.75\%$ or 92%.

Exercise 6

Find the percents for the following test grades. Round your answer to the nearest percent.

A. $\frac{33}{42}$

B. $\frac{24}{40}$

C. $\frac{90}{120}$

D. $\frac{100}{110}$

E. $\frac{10}{20}$

Answers to Exercise 6

A. 79%

B. 60%

C. 75%

D. 91%

E. 50%

Topic A: Self-Test

Mark /12 Aim 10/12

A. Solve to find the missing percents.
(4 marks)

i. 12 is _____% of 60.

ii. 15 is _____% of 75.

iii. 8.2 = _____% of 32.8.

iv. What percent of 64 is 48?

B. Problems.
(4 marks)

i. The $140 jacket was on sale for $126. What percent is the savings?

ii. The rent on the apartment went from $320 a month to $400 dollars a month. What percent is this rent increase?

C. Find the percents for the following grades on tests. Round your answer to the nearest whole percent.
(4 marks)

i. $\frac{12}{15}$

ii. $\frac{10}{19}$

iii. $\frac{71}{92}$

iv. $\frac{132}{140}$

Answers to Topic A Self-Test

A.
i. 20%
ii. 20%
iii. 25%
iv. 75%

B.
i. 10% savings
ii. 25% increase

C.
i. 80%
ii. 53%
iii. 77%
iv. 94%

Topic B: Finding a Number when a Percent of it is Given

$$\frac{\text{is (part)}}{\text{of (whole)}} = \frac{\%}{100}$$

In problems when a certain percentage of a number is given, the **missing term is the whole**. You will be told the % and the part (is), and asked to find the whole (of), which is 100%.

Example A

20% of what number is 14?

- The part = 14
- The whole (“what number”) = unknown (call it N)
- The percent = 20%

Write the proportion:

$$\frac{14}{N} = \frac{20}{100}$$

Solve the proportion:

Simplify:

$$\frac{14}{N} = \frac{\cancel{20}\,1}{\cancel{100}\,5}$$

$$\frac{14}{N} = \frac{1}{5}$$

Cross-multiply to solve:

$$14 \times 5 = N \times 1$$
$$70 = N$$

20% of $70 = 14$

Check by finding 20% of 70. The answer should be **14**.

$$\frac{20}{100} = \frac{N}{70}$$
$$140 = 100N$$
$$14 = N$$

Example B

33⅓% of _____ is 60.

- Part = 60
- Whole = N
- Percent = 33⅓% = ⅓

Set up proportion:

$$\frac{60}{N} = \frac{1}{3}$$
$$3 \times 60 = N \times 1$$
$$180 = N$$

33⅓% of 180 is 60.

To check the answer, find 33⅓% of 180. The answer should be **60**.

Exercise 1

Set up the proportion. Do not solve the question.

A. 18 is 50% of what number?

B. 24 is 15% of what number?

C. 200% of what number is 86?

D. 66⅔% of what number= 500?

Answers to Exercise 1

A. $\frac{18}{P} = \frac{50}{100}$

B. $\frac{24}{N} = \frac{15}{100}$

C. $\frac{86}{P} = \frac{200}{100}$

D. $\frac{500}{P} = \frac{66\frac{2}{3}}{100}$ or $\frac{2}{3}$

Exercise 2

Solve the following. Check your answers to see if you set up the proportion correctly.

A. 60 is 75% of what number?

B. 950 is 95% of:

C. 125 = 33⅓% of what number?

D. 120% of _____ is 6.

E. 270 is 100% of what number?

Answers to Exercise 2

A. $\frac{60}{X} = \frac{75}{100}$, 80

B. $\frac{950}{P} = \frac{95}{100}$, $1,000$

C. $\frac{125}{N} = \frac{33\frac{1}{3}}{100}$, 375

D. $\frac{120}{100} = \frac{6}{J}$, 5

E. $\frac{100}{100} = \frac{270}{N}$, 270

Exercise 3

Solve the questions.

A. 480 is 66⅔% of _____.
B. 40% of _____ is 50.
C. 33⅓% of what number is 99?
D. 3 is 150% of what number?
E. 122 is 80% of _____.

Answers to Exercise 3

A. 720
B. 125
C. 297
D. 2
E. 152.5

Solving Problems when the Percent of a Number is Given

Read the problems carefully. More than one step may be needed. Look at the wording so you will recognize problems missing the whole and be able to tell them from problems missing the part.

Exercise 4

Solve the problems. Round money to the nearest cent.

A. When the business was declared bankrupt, all the creditors (people owed money by the business) were paid 55% of the money owed them.
    i. If a creditor received $8,000 from the bankrupt business, how much money had he really been owed?
    ii. How much money did the creditor lose on this business?
B. In telethons and other fund-raising events, records show that about 80% of the money pledged is

actually collected. The local telethon organizers need to raise $12,000. To actually raise $12,000, their goal for pledges should be what amount?

C. The shoe store sold all merchandise at 25% off in a huge clearance sale. They took in $3,500 in the first day of the sale. If the same shoes had been sold at the regular price, how much money would they have taken in? (Note—this problem has 2 steps. The merchandise was 25% off, so it sold for 100% − 25% = 75% of the original price.)

D. A common rate of commission earned by real estate agents is 3½%. If an agent had a gross income of $63,000 from commissions in one year, what was the value of the houses sold?

Answers to Exercise 4

A. i. $14,545.45

ii. $6,545.45

B. $15,000.00

C. $4,666.67

D. $1,800,000

Topic B: Self-Test

Mark /8 Aim 6/8

A. Solve these questions.
(4 marks)

i. 2.5% of what number is 160?

ii. 5 is 4.5% of _____.

iii. 180 is 90% of what number?

iv. 28 is 35% of what number?

B. Solve these problems.
(4 marks)

i. If a bank insists that new house buyers have a cash down payment of 12%, what house price can a couple afford if they have saved a $15,000 down payment?

ii. Jim has really cut down on his smoking. He now smokes 7 cigarettes a day, which he says is only 20% of what he used to smoke. How many cigarettes a day did Jim smoke before he started cutting down?

Answers to Topic B Self-Test

A.
i. $6,400$
ii. 625
iii. 200
iv. 80

B.
i. $125,000
ii. 35 cigarettes per day

Unit 4 Review: More Working with Percent

Review

You have been practising three types of percent problems. You have learned that one proportion can be used to solve all the problems:

$$\frac{\text{is (part)}}{\text{of (whole)}} = \frac{\%}{100}$$

A. **Finding a Percent of a Number**

- You are given the percent and the whole.
- The missing term is the part (call it *N*). $\frac{N}{\text{whole}} = \frac{\%}{100}$

B. **Finding What Percent One Number is of Another**

- You are told the part and the whole.
- The missing term is the percent (call it *P*). $\frac{\text{part}}{\text{whole}} = \frac{P}{100}$

C. **Finding a Number When a Percent of it is Given**

- You are given the part and the percent.
- The missing term is the whole (call it *W*). $\frac{\text{part}}{W} = \frac{\%}{100}$

Real-life situations and real math problems often require several steps to collect and organize all the information. Look for those extra steps in the problems that follow. When you read the problems look for the part, the whole and the percent. Decide which term is missing. Once you know which term is missing the problem can be solved by using the proportion $\frac{\text{part}}{\text{whole}} = \frac{\%}{100}$ or the appropriate short method.

Questions

Solve these problems using proportion. **Write all your work with the problem so your instructor can help you should you have any difficulty**. Remember to check that the answer makes sense and to write a sentence answer. For these problems, round your answers this way:

- Percents to the nearest tenth of a percent
- Money to the nearest cent
- Decimal fractions to the nearest thousandth

1. You need a minimum of 80% on the test which means you must get at least 36 marks. What are the total possible marks on the test?
2. Ann and Joe bought their home twelve years ago and have paid 45% of the principle amount of their mortgage. They have paid $18,000 towards the principle of the mortgage. What was the principle amount of the mortgage to start with?
3. The waiters at the restaurant must contribute money to be shared among the cocktail servers and kitchen staff. Each waiter contributes 4% of his or her total food and drink sales. Craig's total sales were $645 in his 5 hour shift.
    a. How much money must he contribute to the kitchen and cocktail servers?
    b. Craig made 12% in tips on his sales tonight. What amount were his tips?
4. The cost of hydroelectric power for our home last year was 210% of what it was six years ago. Last year our power bill totalled $960. How much was it six years ago?
5. 16⅔% of the tickets for the rock concert were sold in the first hour the telephone order lines were open. In that hour, 2,500 tickets were sold. What was the total number of tickets available for the concert?
6. The total student enrolment in the school district has increased from 18,506 students to 19,724 students in the last year. What is the percent of this increase in student enrolment?
7. Al bought his secondhand off-road motorcycle for $1,500 and sold it three years later for $1,175. By what percent did his motorcycle depreciate (decrease in value)?
8. Pat operates a street-vendor's cart selling hot dogs, sausages on a bun and soft drinks. The basic pay is $100 per week and 28% commission on all sales over $450 in a week. Pat sold $1,244 of food and soft drinks last week. Calculate Pat's earnings from the street-vending cart for the week.
9. The 1,500 blouses purchased by the large retail chain of ladies' clothing stores cost the company a total of $24,000. The blouses were then priced to sell at $45 each. What is the percent of the mark-up on these blouses? (Hint: First calculate the company's cost price for each blouse.)

10. Maureen was happy to see that she got 27 out of 30 on her English essay. What percent did she get?
11. The ski jackets were on the summer clearance rack marked “45% off”.
    a. What is the sale price of a jacket priced regularly at $229.95?
    b. What is the total cost of this jacket with H.S.T (12%)?
12. Tuition fees at the university have increased from $49 per credit hour to $62 per credit hour in the last three years. What is the percent of the tuition increase?
13. Jill bought 3 bottles of liquor in the US. The bill, including US sales tax, was $28.50. Assume the American dollar was $1.08 Canadian. Calculate:
    a. The value of the liquor in Canadian funds.
    b. Duty at 110%.
    c. HST at 12% (on Canadian value + duty).
    d. Total cost in Canadian funds.
14. The consignment store will sell your good used women’s clothing for you. The store owners take a percentage of the selling price as their fee for service. The consignment charges (the % the store owners keep) are as follows:

Coats	45%
Dresses and skirts	$33\frac{1}{3}\%$
Bridal and evening gowns	50%
Blouses, jeans, and slacks	25%

Lisa and her daughters did a huge closet clean-out and had the following items sold at the consignment store. For each category, calculate the amount for the store fee and the amount Lisa and the girls received.

	Item	Selling Price	Store Fee	Amount for Lisa & Her Daughters
a	Wedding dress	$275		
b	3 dresses at $40 each	3 @ $40 = $120		
c	Lisa’s winter coat at $120	$120		
d	4 pairs of outgrown jeans at $10 each	4 @ $10 = $40		

15. You have just completed four units of the five units in this book. What percent of the book

have you completed?!?!?

Answers to Unit 4 Review

1. 45 marks
2. $40,000
3. a. $25.80
   b. $77.40
4. $457.14
5. 15,000 tickets
6. 6.6% increase
7. 21.7% depreciation
8. $322.32
9. 181.3% mark-up
10. 90% correct
11. a. $126.47
    b. $141.65
12. 26.5% increase
13. a. $30.78
    b. $33.86
    c. $7.76
    d. $72.40
14.

	Item	Selling Price	Store Fee	Amount for Lisa & Her Daughters
a	Wedding dress	$275	$137.50	$137.50
b	3 dresses at $40 each	3 @ $40 = $120	$40.00	$80.00
c	Lisa's winter coat at $120	$120	$54.00	$66.00
d	4 pairs of outgrown jeans at $10 each	4 @ $10 = $40	$10.00	$30.00

15. 80%. Well done!

TEST TIME!

Ask your instructor for the Practice Test for this unit.

Once you've done the Practice Test, you need to do the Unit 4 test.

Again, ask your instructor for this.

Good luck!

Unit 5: Statistics

Introduction and Vocabulary

The word *graph* comes from a Greek word meaning to write or draw.

Graphs are a special type of drawing or picture showing how numbers relate to each other. Graphs are a convenient way to organize numbers. You may know the old saying, “One picture is worth a thousand words.” Graphs give us a general picture of the information to look at first. The details of the information can then be read from the graph.

In this unit you will practice reading five types of graphs: line graphs, bar graphs, histograms, picture graphs, and circle graphs. You will also learn about reading charts and tables.

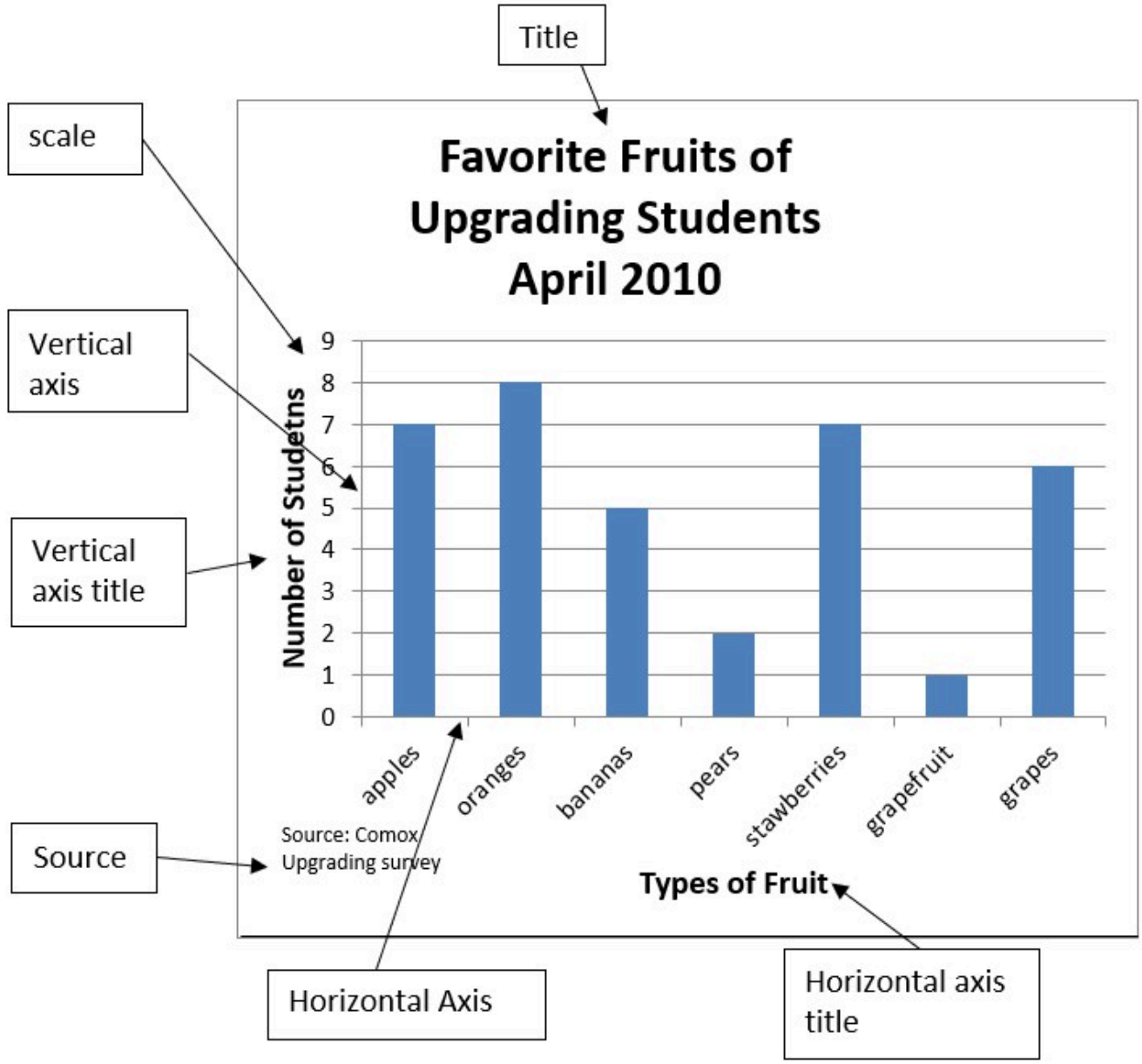
Study this vocabulary:

- **Axis:** An axis is any straight line used for measuring or as a reference. In graphs, we talk about two axes (axes is the plural of axis).
    - The **horizontal axis** goes across the page. (You will learn in later math courses that the horizontal axis is the x-axis.)
    - The **vertical axis** goes up and down the page. (The vertical axis is the y-axis.)

Here is a trick to remember the difference between horizontal and vertical:

- The horizon is that line of earth you see as you look out at a view.
- The word horizontal comes from the root word horizon. The horizon goes from left to right, so does the horizontal line in a graph.

- **Titles:** Titles of a graph give important information about the graph and often tell more about what the scale means.
- **Scale:** Each axis has a scale. The scale is a series of numbers; beside or below the numbers will be a few words telling what the numbers represent. The numbers we want to place on the graph can be read from the scales. The horizontal scale describes the positions on the horizontal (x) axis. The vertical scale describes the positions on the vertical (y) axis.
- **Source:** Graphs often give information that has been collected from other reports or publications. The source of the information should be written under the graph. Knowing where the information has come from helps you decide the accuracy of the information in the graph.
- **Trend:** The trend is the general direction of events, the general idea of changes that are occurring. For example, there is an upward trend in the cost of houses.



[Image Description]

Image Descriptions

Bar Graph Image: A bar graph showing the favourite fruits of upgrading students. The graph is labelled with the following:

- Title: Favourite Fruits of Upgrading Students April 2010
- Vertical axis title: Number of students
- Vertical axis scale: The numbers 0 to 9
- Horizontal axis title: Types of Fruit
- Horizontal axis: Apples, oranges, bananas, pears, strawberries, grapefruit, grapes

- Source: Comox Upgrading Survey

[Return to Image]

Image Attributions

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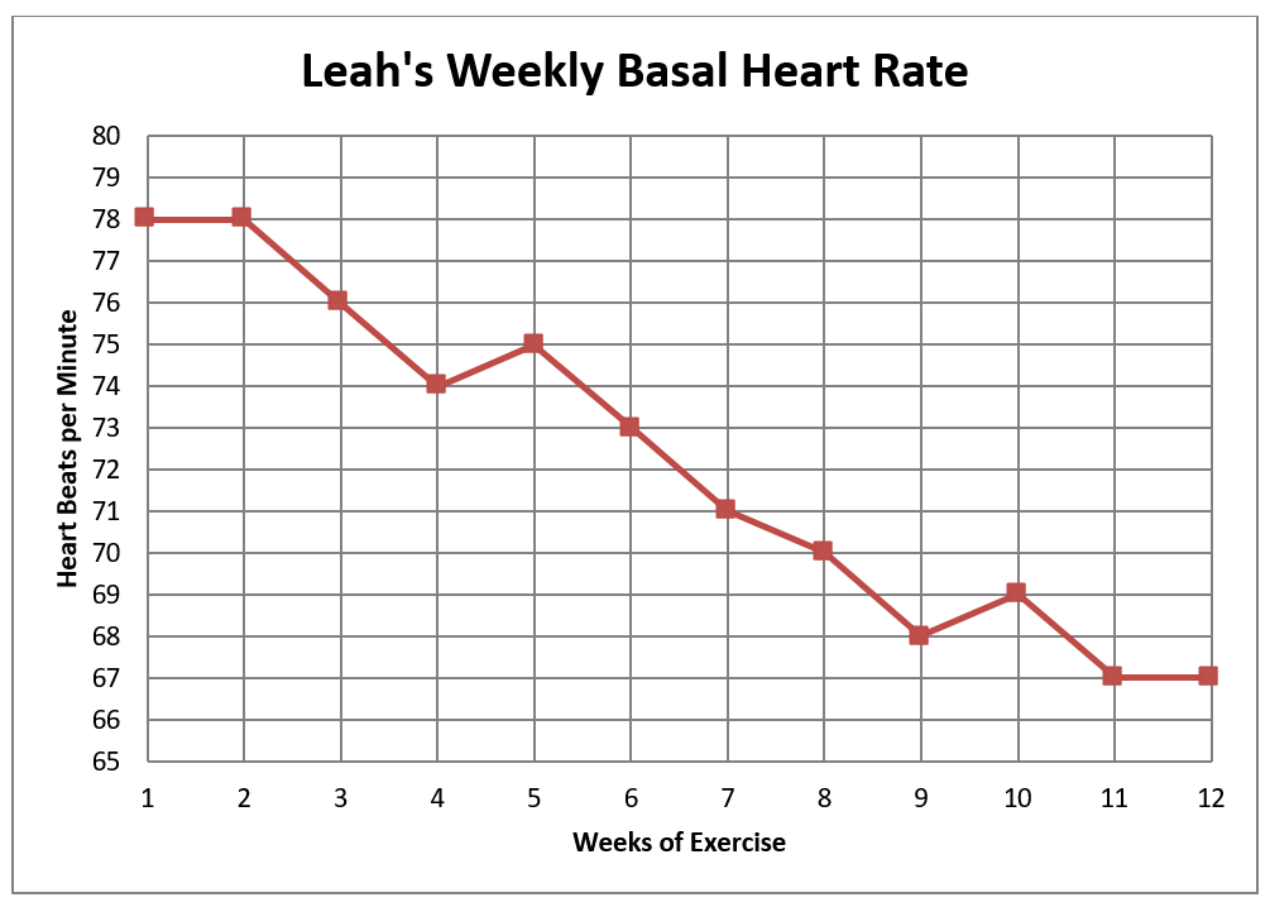
Topic A: Line Graphs

Line graphs are used to show **changes that happen over a period of time**. Line graphs easily show trends and patterns.

The most common way to set up a line graph is to put **time** on the horizontal (x) axis. Whatever is being measured is then put on the vertical (y) axis.

Graph 1



[Image Description]

Leah has taken up cycling and jogging to improve her cardiovascular fitness and for weight control. She takes her pulse every Monday morning before she gets out of bed; that is her basal heart rate. Graph One records Leah's heart rate for the first 12 weeks of her exercise program.

A. What is the title of the graph?

B. What is being measured on the vertical axis?

On this particular graph the **vertical scale** is one heartbeat per line. The scale on the **horizontal axis** is one week per line.

C. What was Leah's basal heart rate on the fourth week of her exercise program?

- Find Week 4 on the horizontal (x) axis.
- Look straight up from Week 4 until you come to the point in the graphed line.
- Now look at the scale on the vertical (y) axis. Lay a ruler or straight piece of paper across the graph to help you read the scale at the point for Week 4.

Leah's basal heart rate was 74 beats/min in week four.

D. Find her basal heart rate in Week 7.

E. What does the graph show us happened between Week 9 and Week 10?

F. What **trend** does this graph show?*The graph shows Leah's basal heart rate is "going down" or decreasing.*

Often, you need to estimate the value of the point in the graphed line. Look at Graph Two which has Leah's same heart rates recorded.

Answers to Graph 1

A. Leah's Weekly Basal Heart Rate

B. Heart beats per minute

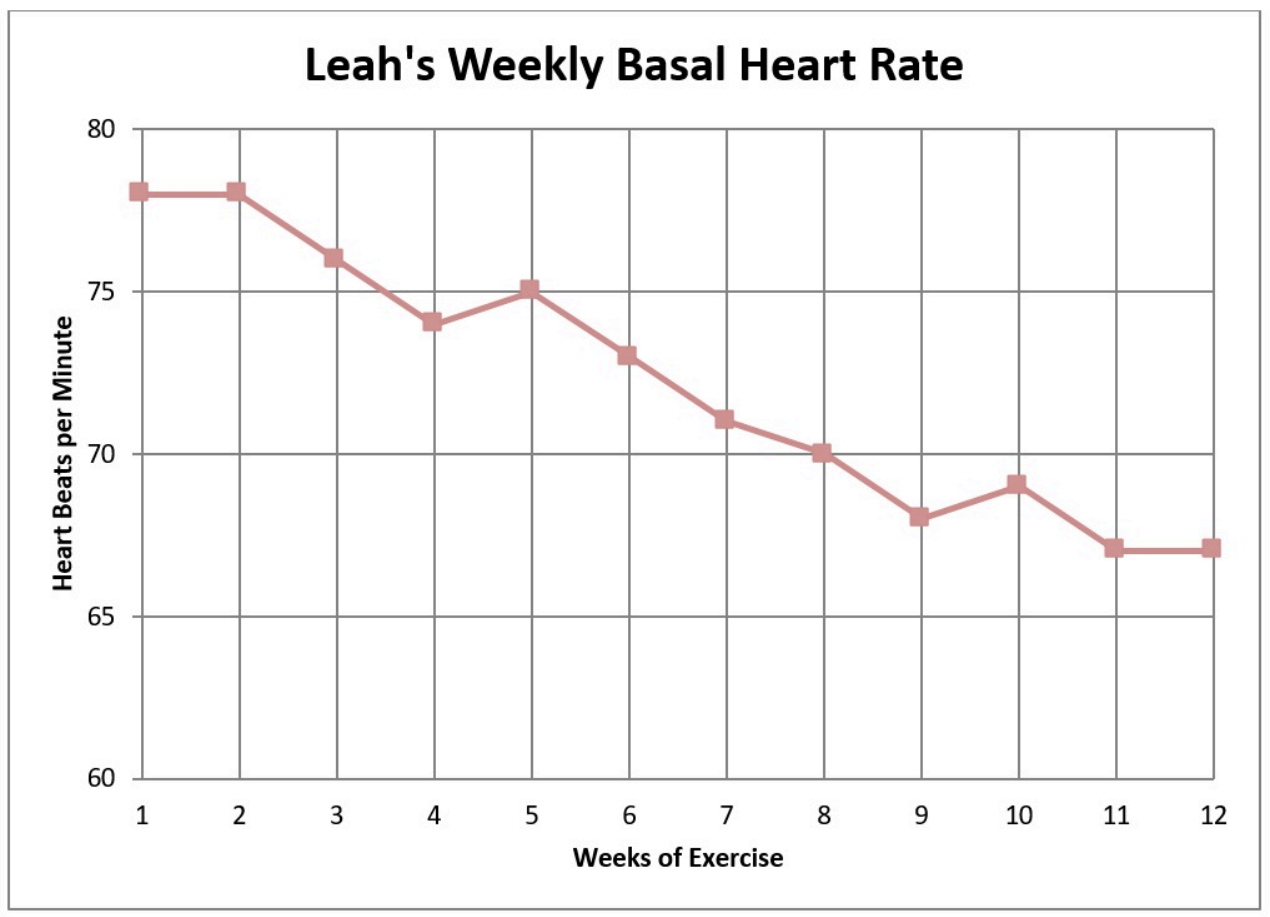
C. Given: 74 beats/min

D. 71 beats/min

E. Leah's heart rate went up from 68 to 69 beats/min

F. Given: Leah's basal heart rate is "going down" or decreasing

Graph 2



[Image Description]

Now the scale on the vertical axis is five heartbeats per line. Use a straightedge (ruler or paper) across the graph to help you read the vertical scale.

A. Give Leah's heart rate in Week 5.

B. Give Leah's heart rate in Week 10.

C. What was Leah's heart rate in Week 12?

D. Using the information on the graphs, tell how much Leah's basal heart rate decreased (in beats per minute) from Week 1 to Week 12.

Answers to Graph 2

A. 75 beats/min

B. 69 beats/min

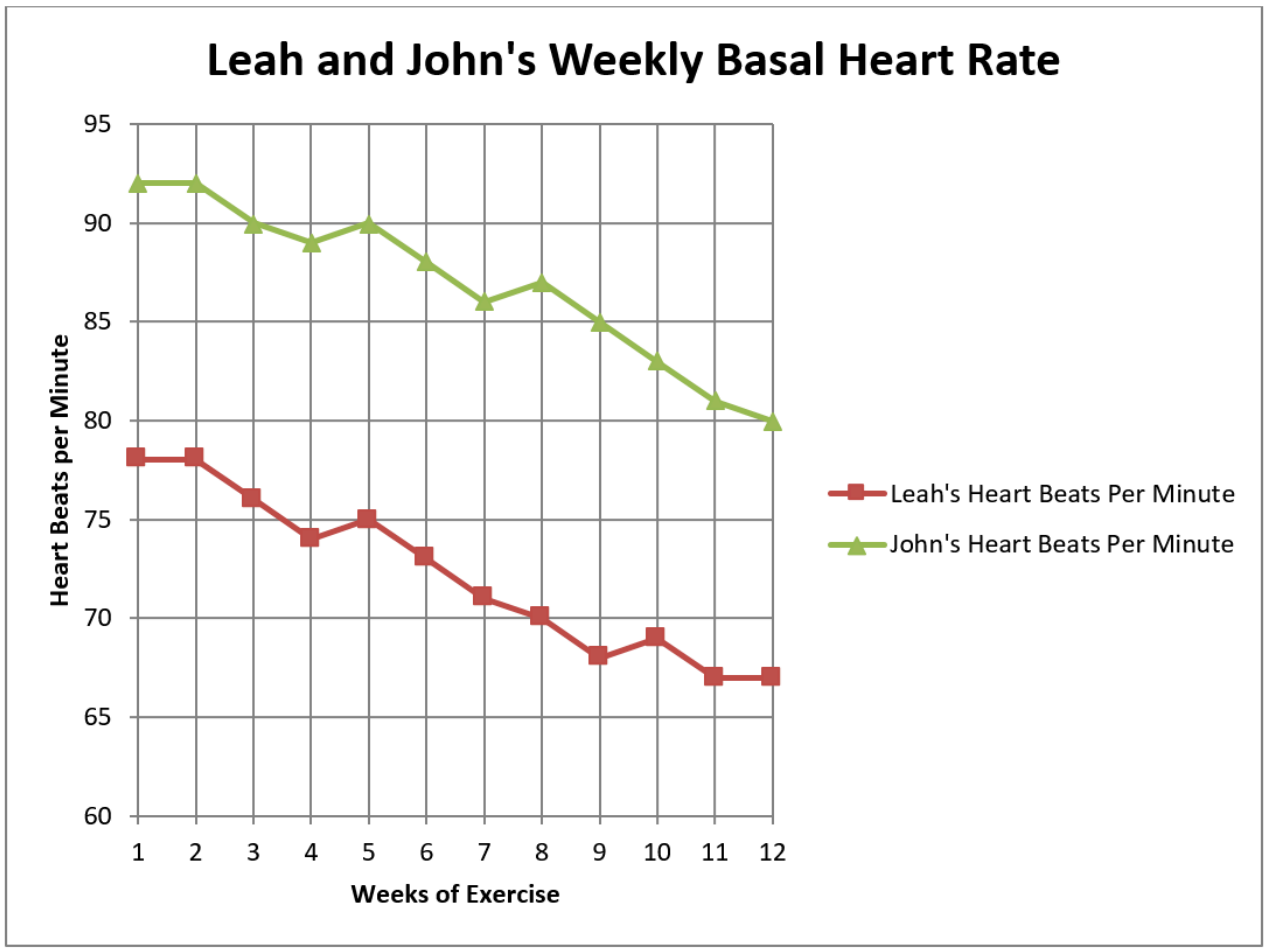
C. 67 beats/min

D. Decreased 11 beats/min (78 – 67 = 11)

The person drawing a graph decides how to label it and how to write the scale on the axes depending on the information to be shown on the graph. The graphs about Leah focus on the range of **her** heart rate. There is no need to make the heart rate scale lower than 55 or higher than 80 for Leah.Graphs often show information about several things on the same graph. Such graphs are very useful for making comparisons. Look for a **legend** or **key** that explains what each graphed line represents. The legend may be printed right by the graphed information or it may be beside or below the graph.

Graph 3

Leah's husband John decided he would exercise as well. He had a rapid heart rate at the beginning of the exercise program. Since his heart rate is higher than 85, we must increase the numbers on the vertical scale so we can graph John's heart rate on the same graph as Leah's.



[Image Description]

A. What was John's heart rate in Week 6?

B. What was John's heart rate in Week 10?

C. How much lower was Leah's heart rate than John's in Week 8?

D. How much did John's heart rate drop in the 12-week exercise program?

E. What was the amount of change in John's heart rate between Week 3 and Week 7? Was it an increase (+) or decrease (-)?

F. Compare the two graphed lines.

   i. How is the slant of the lines different?

   ii. How are the lines the same?

   iii. You can tell from looking at this graph that John and Leah had a similar trend in the change in their basal heart rates. Did their heart rates increase or decrease?

Answers to Graph 3

A. 88 beats/min

B. 83 beats/min
C. 16 beats/min
D. 12 beats/min
E. Decrease 4 beats/min

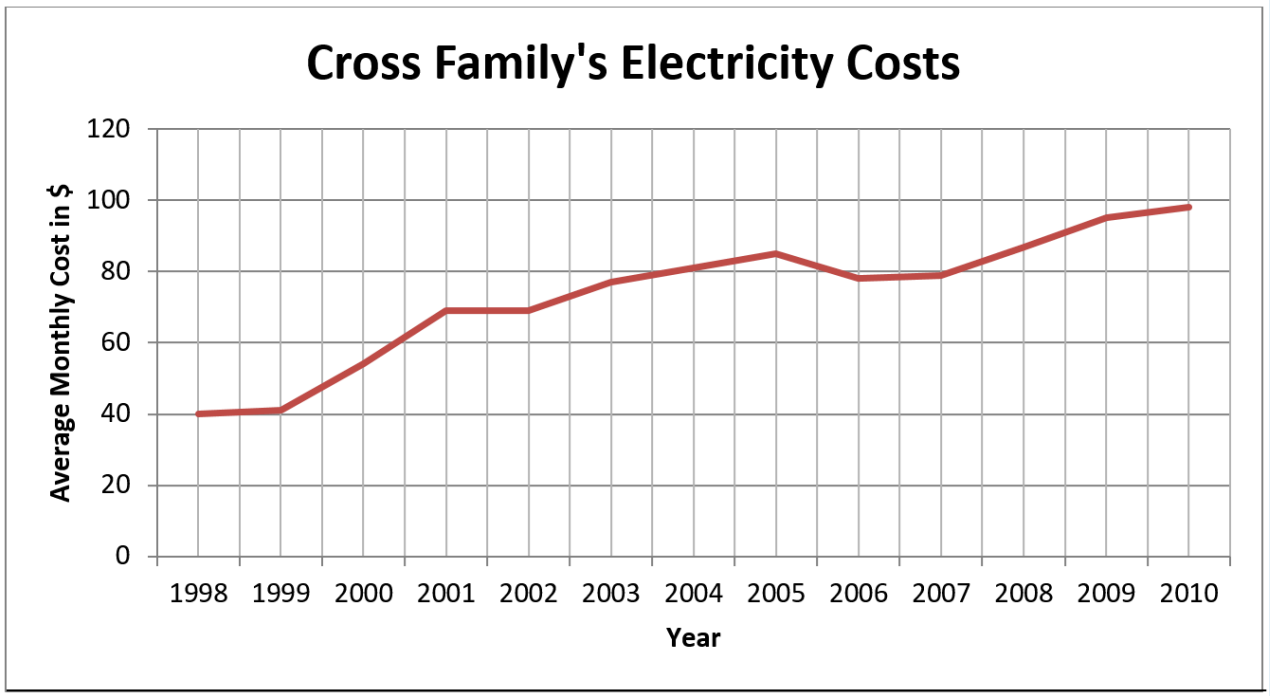
i. John's goes up at Week 8, Leah's goes up at Week 10, Leah stays the same from Week 11 to 12 while John's goes down.
ii. Similar because both decrease at a similar slant and both show an increase in two separate weeks.
iii. Both heart rates decreased.

If the graphed lines have about the same slant, the **rate of change** is the same for the information being graphed. By looking at graphed lines you can tell if one has increased or decreased more quickly. You can easily compare changes and tell when the changes occurred.

Steps to Follow When Reading Line Graphs

1. Read all the titles so you know what the graph is about.
2. Look at the information on the vertical and horizontal scales.
3. Decide what the graphed lines represent. Look for a legend and be sure you know which line is which.
4. Interpret (read) the information on the graph.
    - First, get a general look at what is on the graph.
    - Second, look for the detailed information that you need.

Graph 4



[Image Description]

A. What is the general trend in the Cross's average monthly cost for electricity?
B. What was the average monthly cost of electricity in 1998?
C. How much more did they pay per month in 2003 than in 2000?
D. What happened to the average cost between 2005 and 2006?
E. By how much did the Cross's average monthly electricity cost increase between 1998 and 2010?

Answers to Graph 4

A. Increased
B. $40
C. Approximately $23 ($77 – 54)
D. Decreased
E. $58

Line graphs can also be used to graph two different types of related information on the same chart. For example, we may have wanted to put Leah and John's weight changes and heart rate changes on the same graph.

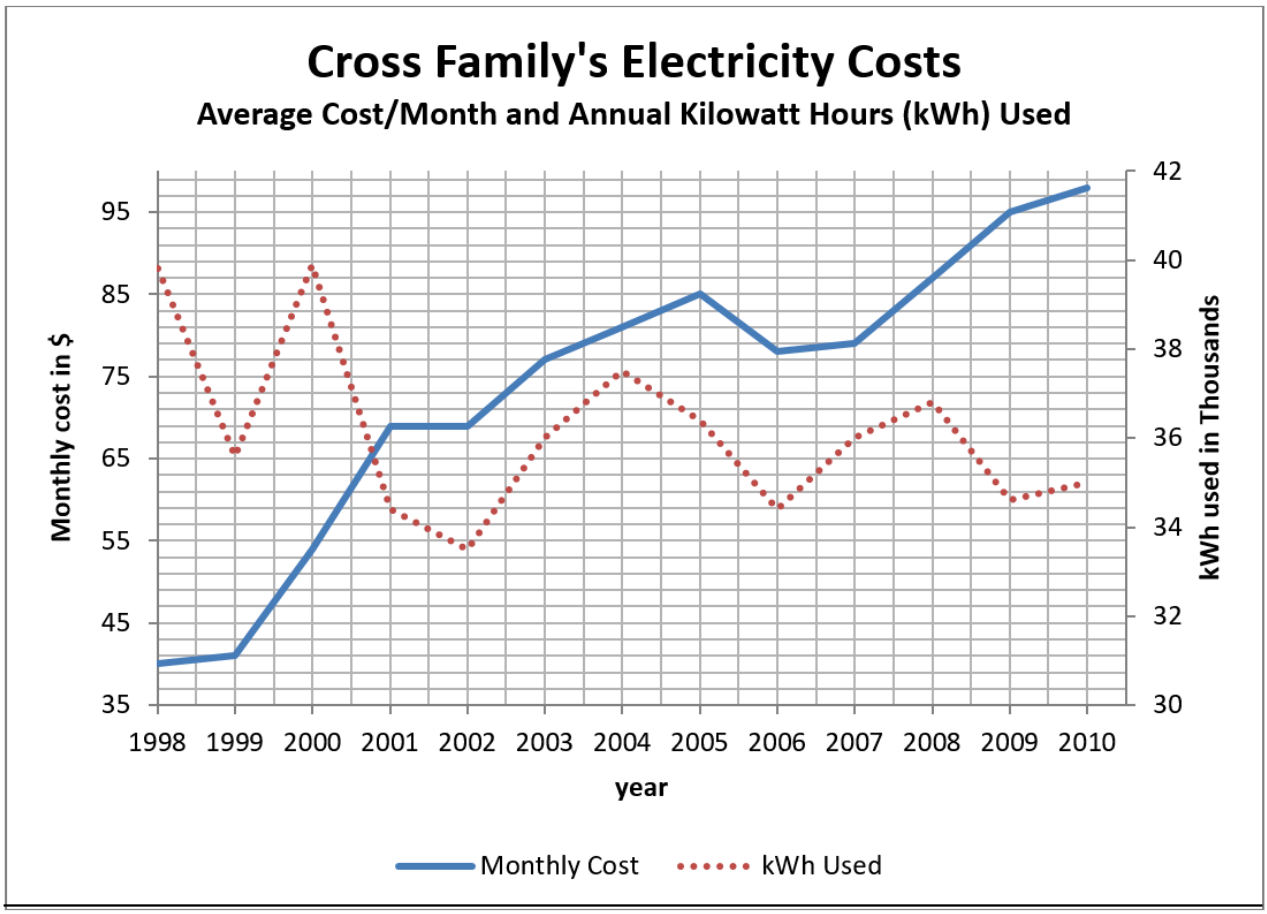
When we graph different types of information,

- A sub-title usually explains the two types of information (written in smaller print under the title).
- The horizontal axis is the same.
- The vertical axis on the **right** side of the graph has the scale for the second set of information. For example, on the graph for Leah and John the right vertical axis would be marked in kilograms for weight changes.

Steps to Follow When Reading Line Graphs With Two Types of Information

1. Read the titles and subtitles.
2. Look at the legend to identify the graphed lines and check where the scale for each line is written (on the left or on the right vertical axis).
3. Go up from the horizontal axis in the same way, and read the appropriate scale on the left or right vertical axis for each graphed line.

Graph 5



[Image Description]

A. What is the title of this graph?

B. What is the subtitle?

C. Look at the legend. What does the graphed line for "Kilowatt Hours Used" look like?

D. Where is the scale for kilowatt hours?

E. How is the scale for kilowatt hours labeled?

> *in thousands* means that each figure in the scale is to be multiplied by 1,000. 32 means 32,000.

F. Which two years was the use of kWh the greatest?

G. When was the use of kWh the least?

H. Look at 1999 and 2000.

   i. Find the amount of kWh used (in thousands) in 1999 _____ and the cost of electricity in 1999 _____.

   ii. Compare to the amount of kWh used in 2000 _____ and the cost of electricity in 2000 _____.

   iii. What can you conclude by comparing the cost and the kWh use in 1999 and 2000?

I. Now look at 2001 and 2002.

   i. Compare the use of kWh's.
   2001 _____2002 _____

   ii. Compare the cost.
   2001 _____2002 _____

   iii. What can you conclude?

J. Between 2009 and 2010 the use of kWh went*up or down*? by _____ kWh.The cost of electricity went *up or down*? by _____ $. .

Answers to Graph 5

A. Cross Family's Electricity Costs

B. Average Cost/month & Annual Kilowatt Hours Used

C. Dotted line

D. Right side of graph

E. kWh used in thousands

F. 1998 + 2000

G. 2002

H.
   i. ~36,000 kWh and $41.

   ii. ~40,000 kWh and $54.

   iii. The price per kWh increased.

I.
   i. 2001: ~34,500 kWh; 2002: ~33,500 kWh

   ii. 2001: $69; 2002: $69

   iii. You can conclude the price/kWh increased.

J. The use of kWh went up by ~500 kWh. Cost went up by $3.

The steepness of the slant of a graphed line gives you a picture of the **rate of change**. The steeper the slant the greater the change.

This double graph shows that the average monthly cost of electricity has increased while the annual use of kilowatt hours has shown an overall decrease.

Image Descriptions

Graph 1 (Line Graph)

A line graph shows the change in Leah's weekly basal heart rate over 12 weeks of exercise.

- The vertical axis is heart beats per minute and has the numbers 65 to 80 in increments of 1.
- The horizontal axis is weeks of exercise and has the numbers 1-12 in increments of 1.

The line graph data is represented in the following table:

Leah's Weekly Basal Heart Rate

Weeks of Exercise (Horizontal Axis)	Heart Beats per Minute (Vertical Axis)
1	78
2	78
3	76
4	74
5	75
6	73
7	71
8	70
9	68
10	69
11	67
12	67

[Return to Image]

Graph 2 (Line Graph)

A line graph showing Leah's weekly basal heart rate.

- The vertical axis is heart beats per minute and has the numbers 60 to 80 in increments of 5.
- The horizontal axis is weeks of exercise and has the numbers 1-12 in increments of 1.

The line graph data is represented in the following table:

Leah's Weekly Basal Heart Rate

Weeks of Exercise (Horizontal Axis)	Heart Beats per Minute (Vertical Axis)
1	78
2	78
3	76
4	74
5	75
6	73
7	71
8	70
9	68
10	69
11	67
12	67

[Return to Image]

Graph 3 (Line Graph)

A line graph showing Leah and John's weekly basal heart rate.

- The vertical axis is heart beats per minute and has the numbers 60 to 95 in increments of 5.
- The horizontal axis is weeks of exercise and has the numbers 1-12 in increments of 1.
- The legend refers to two different graphed lines on the line graph – one refers to Leah's heart beats per minute, and the other refers to John's heart beats per minute.

The line graph data is represented in the following table:

Leah and John's Weekly Basal Heart Rate

Weeks of Exercise (Horizontal Axis)	Leah's Heart Beats per Minute (Vertical Axis)	John's Heart Beats per Minute (Vertical Axis)
1	78	92
2	78	92
3	76	90
4	74	89
5	75	90
6	73	88
7	71	86
8	70	87
9	68	85
10	69	83
11	67	81
12	67	80

[Return to Image]

Graph 4 (Line Graph)

A line graph showing the Cross family's electricity costs.

- The vertical axis is the average monthly cost in dollars, and contains the numbers 0 to 120 in increments of 20.
- The horizontal axis is years and contains the years 1998-2010 in increments of one year.

The line graph data is represented in the following table:

Cross Family's Electricity Costs

Year (Horizontal Axis)	Average Monthly Cost in $ (Vertical Axis)
1998	40
1999	41
2000	54
2001	69
2002	69
2003	77
2004	81
2005	85
2006	78
2007	79
2008	87
2009	95
2010	98

[Return to Image]

Graph 5 (Line Graph)

A line graph showing the Cross family's electricity costs, displaying both average cost/month and annual kilowatt hours (kWh) used.

- The left vertical axis is the monthly cost in dollars, and contains the numbers 35 to 99 in increments of 2.
- The right vertical axis is the kWh used in the thousands, and contains the numbers 30 to 42 in increments of 2.
- The horizontal axis is years and contains the years 1998-2010 in increments of one year.
- The legend refers to two different graphed lines on the line graph – one refers to the monthly cost, and the other refers to the kWh used.

The line graph data is represented in the following table:

Cross Family's Electricity Costs: Average Cost/Month and Annual Kilowatt Hours (kWh) Used

Year (Horizontal Axis)	Monthly cost in $ (Left Vertical Axis)	kWh used in Thousands (Right Vertical Axis)
1998	40	~40
1999	41	~36
2000	54	~40
2001	69	~34.5
2002	69	~33.5
2003	77	~36
2004	81	~37.5
2005	85	~36.5
2006	78	~34.5
2007	79	~36
2008	87	~37
2009	95	~34.5
2010	98	~35

[Return to Image]

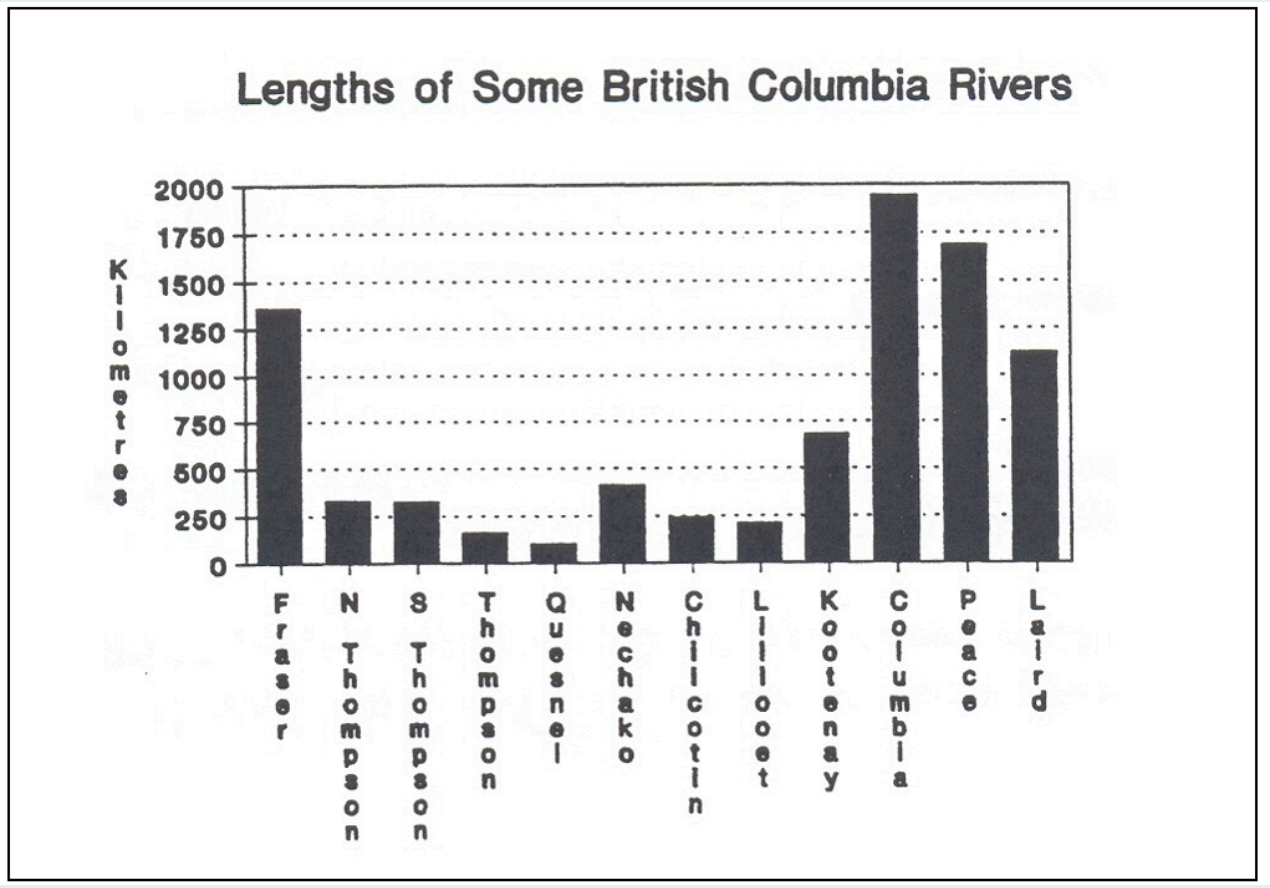
Topic B: Bar Graphs

Bar graphs **compare quantities**. Bar graphs are commonly used to illustrate information in newspapers, in magazine articles, and so on. Bar graphs may be written with the bars arranged vertically or horizontally. Graph One is shown both ways – first with vertical bars and second with horizontal bars.

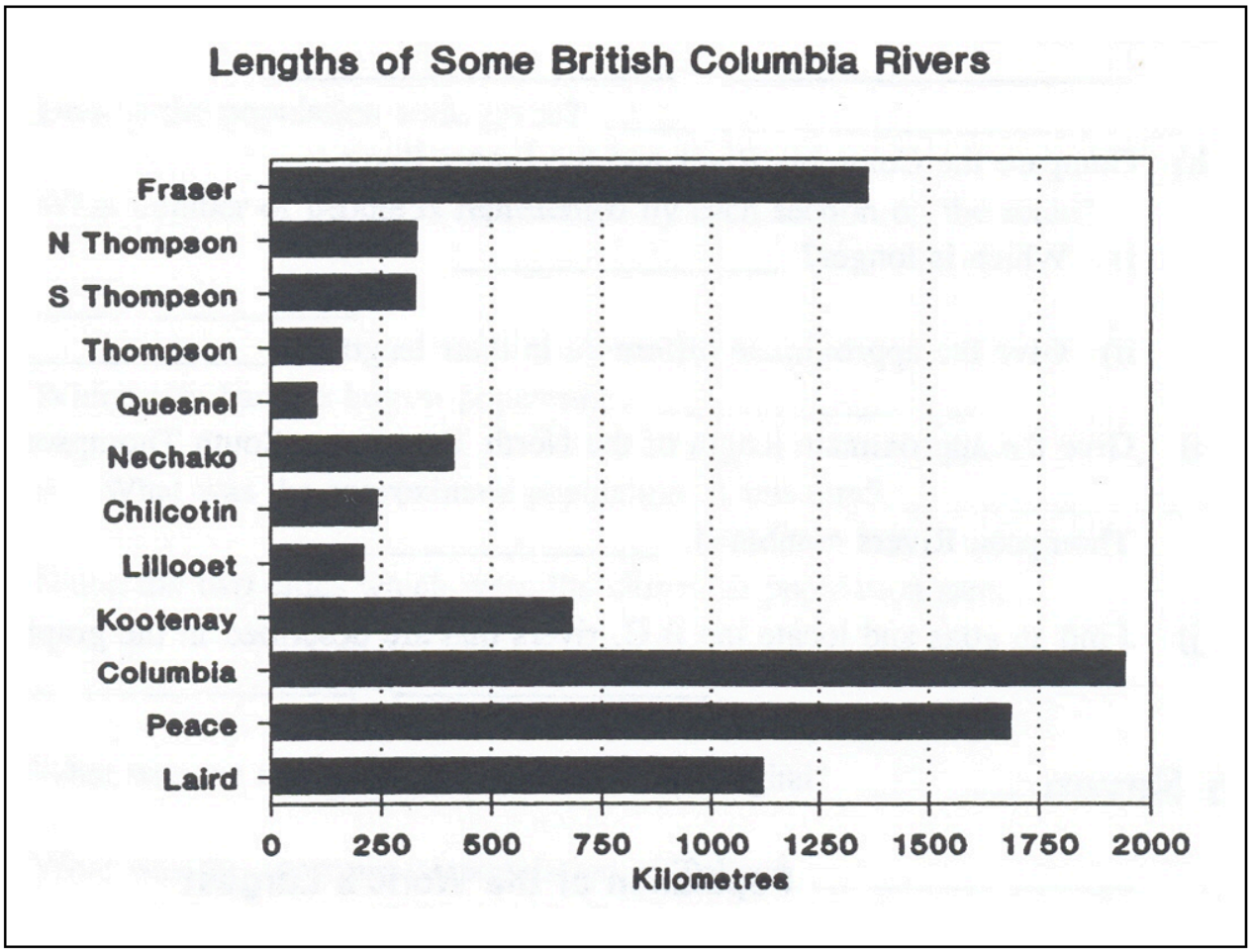
Steps to Follow When Reading a Bar Graph

1. Read the title and subtitles so you know what you are looking at.
2. Read the information on the vertical and horizontal axes. Notice that each bar represents a different item.
3. Look carefully at the scale. What unit of measure is being used? The unit of measure will be the same for each bar so that you can compare them.
4. Compare the length or height of each bar to find the information that you want.

Graph 1



[Image Description]



[Image Description]

A. How many rivers are shown on this graph?

B. What is the title of the graph?

C. What is the unit of measure?

D. Look at the scale for kilometres. How many kilometres are represented by each division on the page?

E. Which river is the longest? What is its length?

F. Which river is the shortest? What is its length?

G. Name two rivers which are approximately the same length.

H. Compare the Columbia River and the Fraser River.

   i. Which is the longer?

   ii. Give the approximate difference in their lengths.

I. Give the approximate length of the North Thompson, South Thompson, and Thompson Rivers combined.

Answers to Graph 1

A. 12 rivers

B. Lengths of some British Columbia Rivers

C. Kilometres

D. 250 km

E. Columbia; ~1,950 km

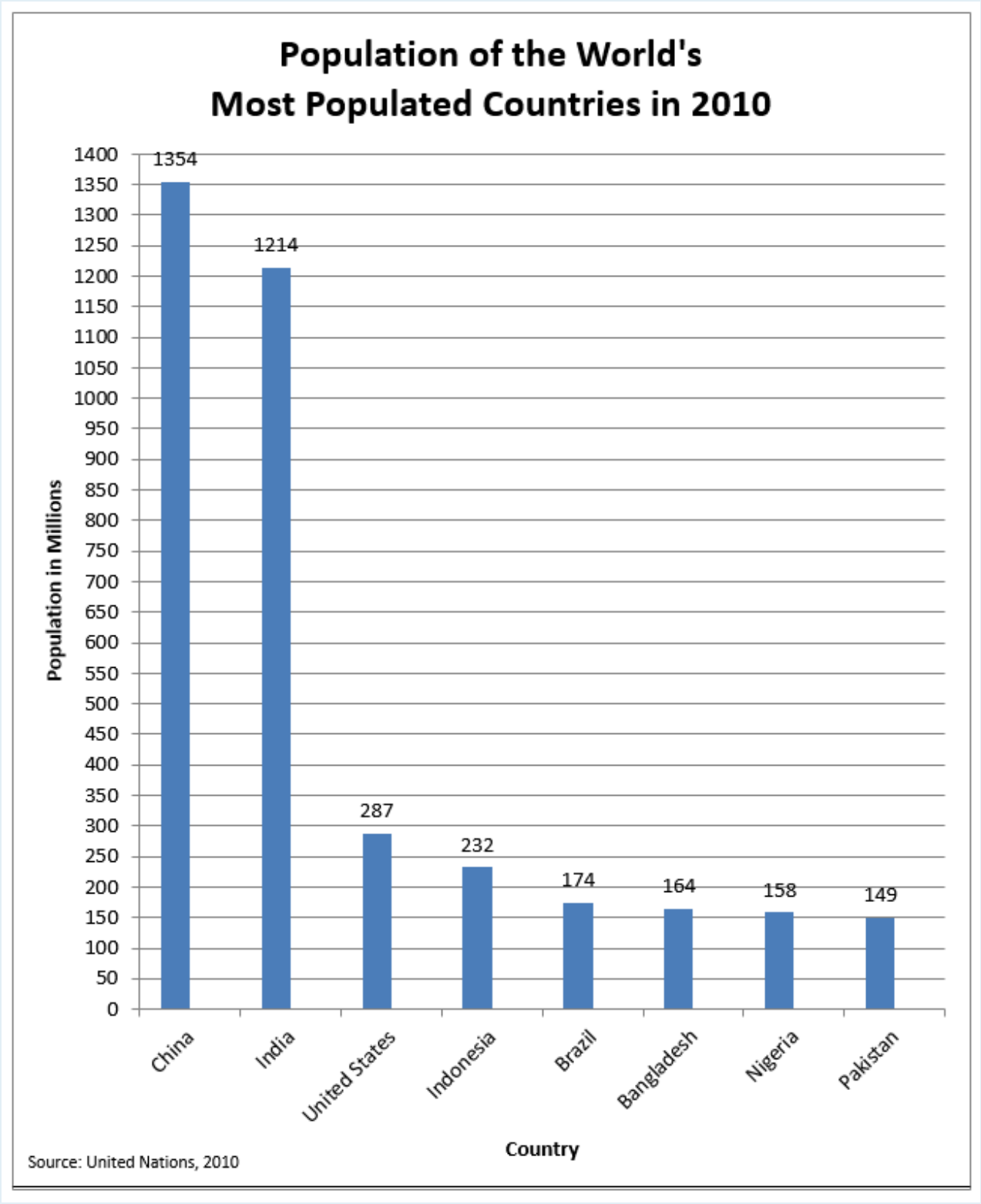
F. Quesnel; ~100 km

G. North Thompson & South Thompson

H.
   i. Columbia
   ii. ~600 km (1,950 – 1,350)

I. ~750 km


Graph 2



[Image Description]

A. Give the source of the information for this graph.

B. What is the unit of measure for the population scale?

C. What number of people is represented by each section on the scale?

D.
   i. Which country had the largest population?
   ii. What was the approximate population of this country?

E. Name the two countries which were the closest in population size.

F. What was the approximate population of Bangladesh?

G. What was the approximate population of India?

Answers for Graph 2

A. United Nations

B. In the Millions

C. 50 million

D.
   i. China
   ii. 1,354,000,000

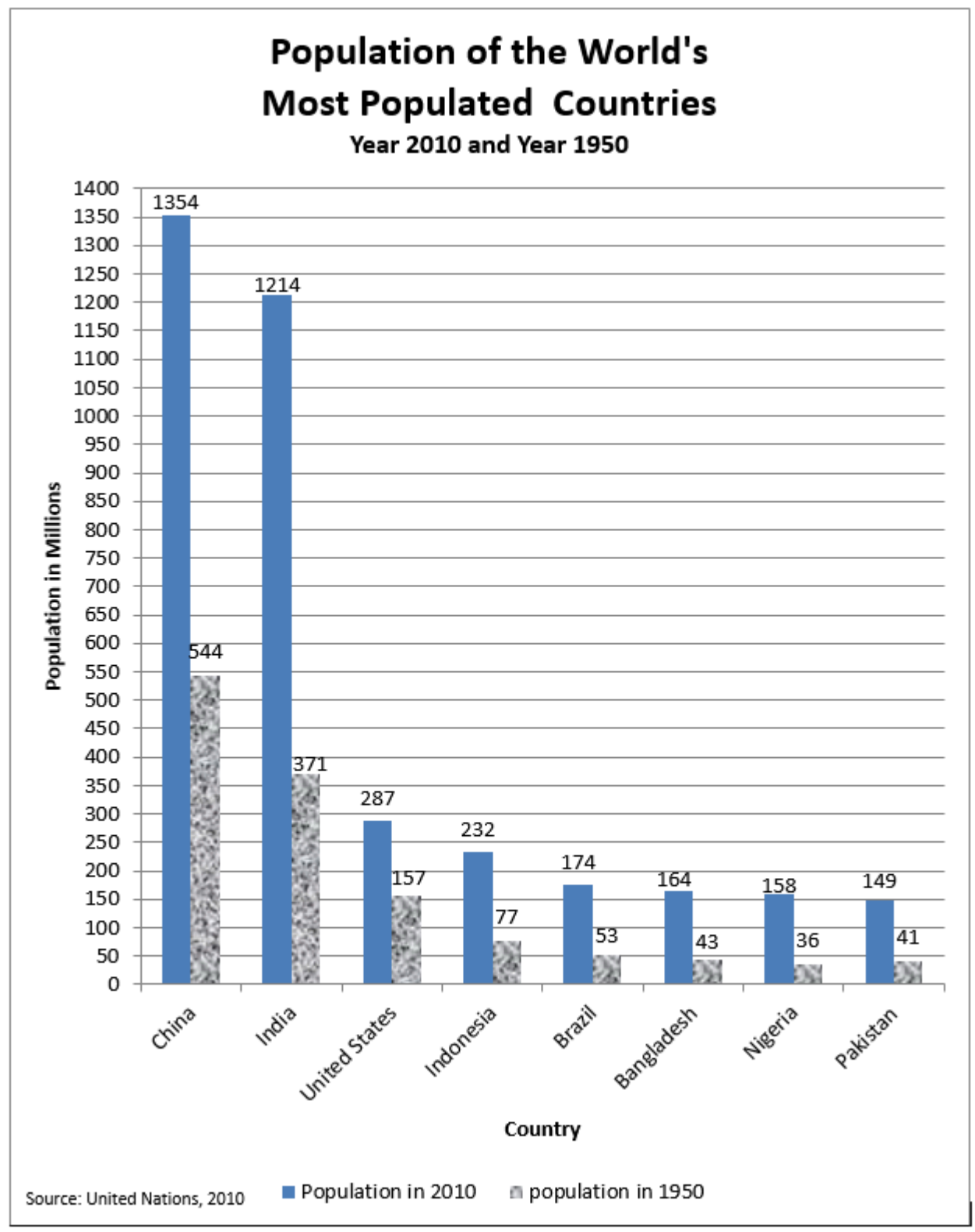
E. Nigeria and Bangladesh

F. 164,000,000

G. 1,214,000,000

Bar graphs can show more than one type of information for each item. These graphs are useful for making comparisons. The bars are usually shaded or coloured differently and a legend will be placed near the graph. The bar graphs must still all use the same unit of measure.

Graph 3



[Image Description]

A. What is the subtitle?

B. Look at the legend. The grey bars give each country's population for what year? The patterned bar gives the population for these same countries in what year?

C. What trend does the graph show?

D.
   i. Which country had the largest increase in population? (this means, which country's population went up by the highest number)
   ii. About how much was that increase?

E.
   i. Which country had the least change in population?
   ii. About how much was that change?

Answers for Graph 3

A. Year 2010 and Year 1950

B. 2010, 1950

C. That countries around the world are growing in population

D.
   i. India
   ii. The increase was 843 million

E.
   i. Pakistan
   ii. The increase was 108 million

Image Descriptions

Graph 1.1 (Bar Graph)

A bar graph showing the lengths of some British Columbia rivers.

- The vertical axis is kilometres and has the numbers 0 to 2,000 in increments of 250.
- The horizontal axis is the names of the following British Columbia rivers: Fraser, North Thompson, South Thompson, Thompson, Quesnel, Nechako, Chilcotin, Lillooet, Kootenay, Columbia, Peace, and Laird.

The bar graph data is represented in the following table:

Lengths of Some British Columbia Rivers

British Columbia Rivers (Horizontal Axis)	Kilometres (Vertical Axis)
Fraser	~1,350
N. Thompson	~300
S. Thompson	~300
Thompson	~150
Quesnel	~100
Nechako	~400
Chilcotin	~250
Lillooet	~200
Kootenay	~675
Columbia	~1,950
Peace	~1,675
Laird	~1,100

[Return to Image]

Graph 1.2 (Bar Graph)

A bar graph showing the lengths of some British Columbia rivers.

- The vertical axis is the names of the following British Columbia rivers: Fraser, North Thompson, South Thompson, Thompson, Quesnel, Nechako, Chilcotin, Lillooet, Kootenay, Columbia, Peace, and Laird.
- The horizontal axis is kilometres and has the numbers 0 to 2,000 in increments of 250.

The bar graph data is represented in the following table:

Lengths of Some British Columbia Rivers

British Columbia Rivers (Vertical Axis)	Kilometres (Horizontal Axis)
Fraser	~1,350
N. Thompson	~325
S. Thompson	~325
Thompson	~150
Quesnel	~100
Nechako	~400
Chilcotin	~250
Lillooet	~200
Kootenay	~675
Columbia	~1,950
Peace	~1,675
Laird	~1,100

[Return to Image]

Graph 2 (Bar Graph)

A bar graph showing the population of the world's most populated countries in 2010.

- The vertical axis is population in millions, and has the numbers 0 to 1,400 in increments of 50.
- The horizontal axis is countries, and contains the following: China, India, United States, Indonesia, Brazil, Bangladesh, Nigeria, and Pakistan.

The bar graph data is represented in the following table:

Population of the World's Most Populated Countries in 2010

Country (Horizontal Axis)	Population in Millions (Vertical Axis)
China	1,354
India	1,214
United States	287
Indonesia	232
Brazil	174
Bangladesh	164
Nigeria	158
Pakistan	149
Source: United Nations, 2010	

[Return to Image]

Graph 3 (Bar Graph)

A bar graph showing the population of the world's most populated countries in 2010 and 1950.

- The vertical axis is population in millions, and has the numbers 0 to 1,400 in increments of 50.
- The horizontal axis is countries, and contains the following: China, India, United States, Indonesia, Brazil, Bangladesh, Nigeria, and Pakistan.
- The legend denotes that each country is associated with two different bars – one referring to its population in 2010, and the other referring to its population in 1950.

The bar graph data is represented in the following table:

Population of the World's Most Populated Countries: Year 2010 and Year 1950

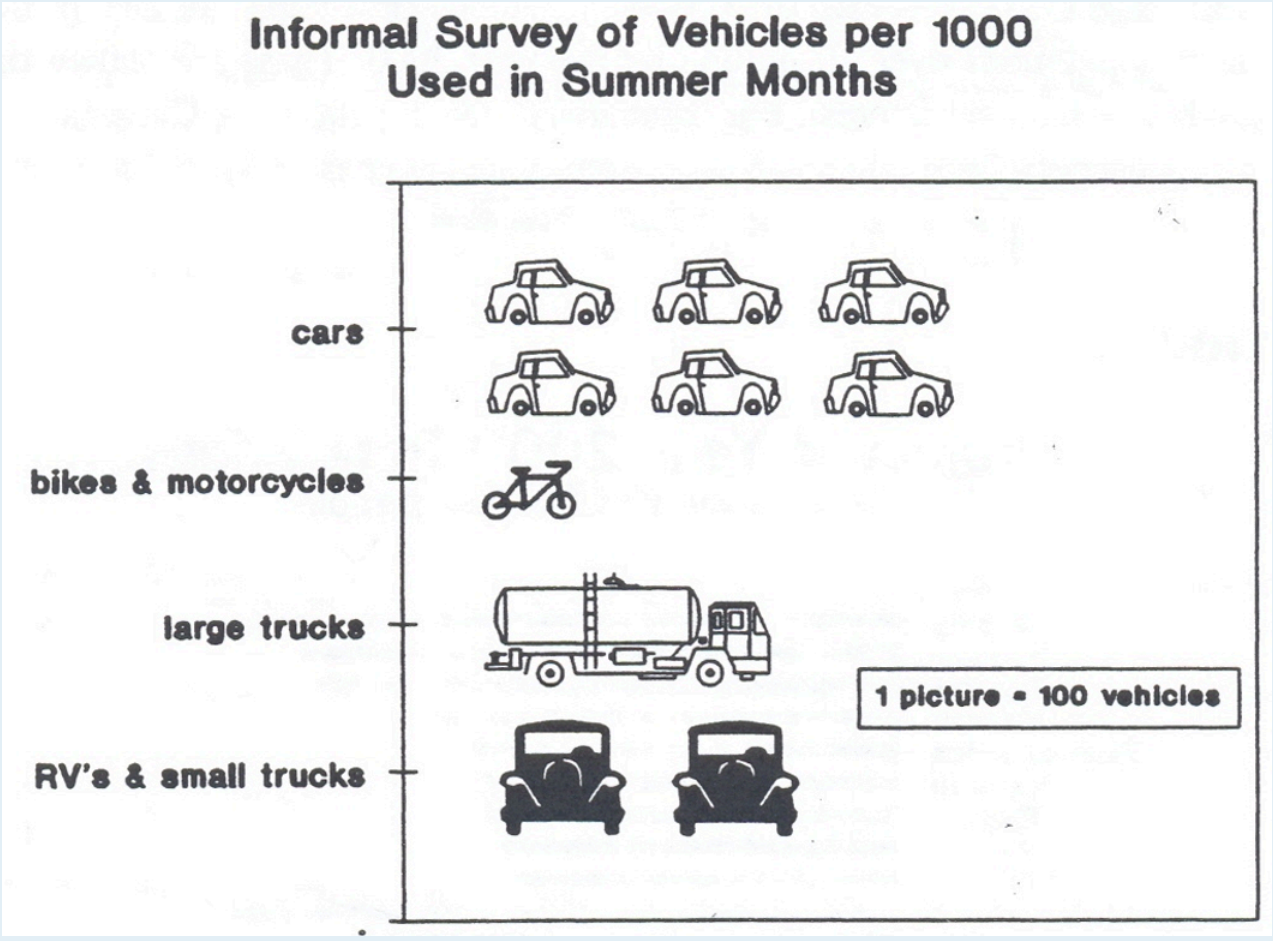
Country (Horizontal Axis)	Population in Millions in 2010 (Vertical Axis)	Population in Millions in 1950 (Vertical Axis)
China	1,354	544
India	1,214	371
United States	287	157
Indonesia	232	77
Brazil	174	53
Bangladesh	164	43
Nigeria	158	36
Pakistan	149	41
Source: United Nations, 2010		

[Return to Image]

Topic C: Picture Graphs

Picture graphs are similar to bar graphs. **Picture graphs show comparisons between quantities**. A little picture represents a certain amount. Look for the legend to find out that amount. Picture graphs will give fractions of a picture also. For example, if the picture represents 100 things, half a picture would be 50.

Graph 1



[Image Description]

A. As a quick first impression when you look at this graph, which type of vehicle is most in use?

B. What does each picture represent according to the legend?

C. Out of every 1,000 vehicles, how many are:

i. cars?
ii. bikes and motorcycles?
iii. large trucks?
iv. RVs and small trucks?

D. Look for other examples of picture graphs in the newspaper and in magazines. Television programs often display picture graphs to illustrate statistics.

Answers for Graph 1

A. car
B. 100 vehicles
C.
i. 600
ii. 100
iii. 100
iv. 200

Image Descriptions

Graph 1 (Picture Graph)

A picture graph showing the results of an informal survey of vehicles per 1,000 used in summer months.

- The vertical axis is vehicles, and contains the following types of vehicles: cars, bikes & motorcycles, large trucks, and RVs & small trucks.
- The horizontal axis is pictures of vehicles, each of which stands for 100 vehicles of the corresponding type according to the legend.

The picture graph data is represented in the following table:

Informal Survey of Vehicles per 1,000 Used in Summer Months

Type of Vehicle (Vertical Axis)	**Number of Pictures (Horizontal Axis)**
Cars	6
Bikes & Motorcycles	1
Large Trucks	1
RVs and Small Trucks	2

[Return to Image]

Topic D: Circle Graphs (“Pie Graphs”)

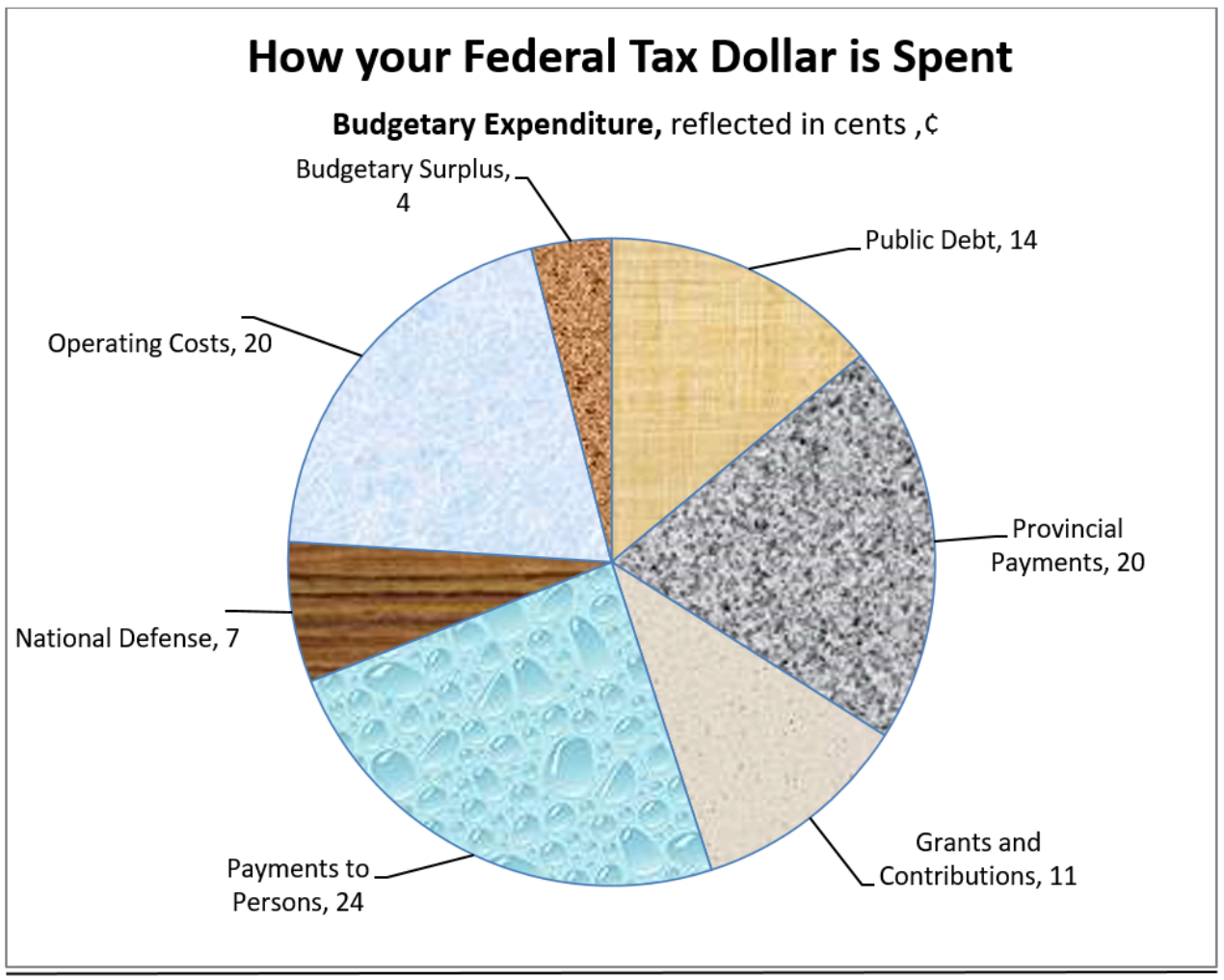
Circle graphs **show how the parts of something compare to each other**. Circle graphs also **give a good picture of each part compared to the whole thing**. In a circle graph or pie graph, the complete circle is the whole thing. The parts of a circle graph may be identified with a percentage. The total of the parts must be 100%.

Graph 1

The circle represents each dollar the government spends. The information for the graph was found at the Department of Finance, April 2010.

Where Your Tax Dollar Goes – 2007-2008 (Department of Finance Canada)

The parts are shown as cents of the dollar.



[Image Description]

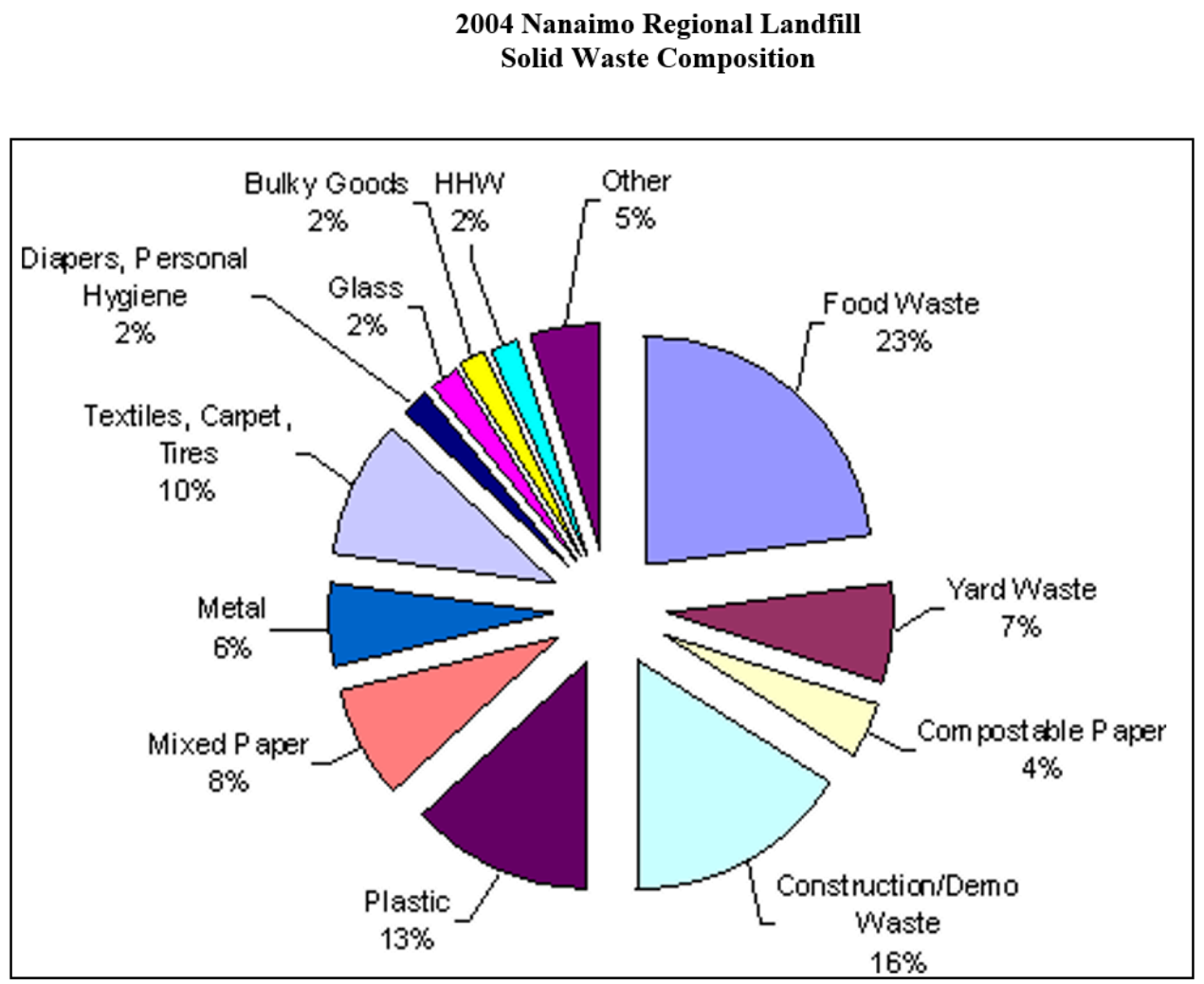
A. What is the biggest expense of the federal government?
B. How much of each federal dollar is spent in actually operating the government business?
C. What part of the federal dollar is spent on defence?
D. How much of each dollar is spent on Provincial Payments? Write this amount as a percent.
E. What is the smallest expenditure of the federal government? Write this amount as a percent.

Answers to Graph 1

A. Payments to Persons
B. 20¢
C. 7¢
D. 20¢; 20%
E. Budgetary Surplus; 4%

Graph 2

2004 Nanaimo Regional Landfill Solid Waste Composition



[Image Description]

A. What makes up the largest part of the waste in the landfill site?

B. What four categories contribute equal weight to the landfill site?

C. In a municipality of 139,000 people, the amount of waste going to a landfill site in one day is 150 tonnes.

   i. What is the mass of plastics?

   ii. What is the mass of yard waste?

   iii. What is the mass of construction/demo waste?

iv. If all the food waste was composted, how many tonnes of waste would *not* end up in the landfill each day?

D. The plastics category can be separated into these categories:

- 6% Non-recyclable mixed plastics
- 4% film plastic
- 3% recyclable rigid food containers

If all the 3% recyclable rigid food containers were actually recycled, how many tonnes of waste would not end up in the landfill?

Answers to Graph 2

A. Food waste

B.
- Diapers, Personal Hygiene
- Glass
- Bulky Goods
- HHW (Household Hazardous Waste

C.
i. 19.5 tonnes
ii. 10.5 tonnes
iii. 24 tonnes
iv. 34.5 tonnes

D. 4.5 tonnes

Image Descriptions

Graph 1 (Circle Graph)

A circle graph showing the Canadian tax dollar was spent in the 2007-2008 fiscal year.

- The whole circle represents one dollar of the Canadian budgetary expenditure.
- Each part reflects one of the specific expenses, reflected in cents to the dollar, ¢. The included parts are (clockwise from top): Public Debt, Provincial Payments, Grants and Contributions, Payments to Persons, National Defense, Operating Costs, and Budgetary Surplus.

The circle graph data is represented in the following table:

How Your Federal Tax Dollar is Spent

Expense	Expenditure, reflected in cents to the dollar, ¢
Public Debt	14
Provincial Payments	20
Grants and Contributions	11
Payments to Persons	24
National Defense	7
Operating Costs	20
Budgetary Surplus	4
Source: Where Your Tax Dollar Goes – 2007-2008 (Department of Finance Canada)	

[Return to Image]

Graph 2 (Circle Graph)

A circle graph showing the solid waste composition of the Nanaimo Regional Landfill in 2004.

- The whole circle represents the total composition of solid waste in the Nanaimo Regional Landfill in 2004.
- Each part reflects one element of the total composition. The included elements are (clockwise from top): Food Waste, Yard Waste, Compostable Paper, Construction/Demo Waste, Plastic, Mixed Paper, Metal, Textiles & Carpet & Tires, Diapers & Personal Hygiene, Glass, Bulky Goods, HHW, and Other.

The circle graph data is represented in the following table:

2004 Nanaimo Regional Landfill Solid Waste Composition

Element of Waste	Percentage of Total
Food Waste	23%
Yard Waste	7%
Compostable Paper	4%
Construction/Demo Waste	16%
Plastic	13%
Mixed Paper	8%
Metal	6%
Textiles, Carpet, Tires	10%
Diapers, Personal Hygiene	2%
Glass	2%
Bulky Goods	2%
HHW	2%
Other	5%
Source: 2004 Nanaimo Regional Landfill Solid Waste Composition	

[Return to Image]

Topic E: Histograms

A histogram is a special bar graph that shows how a frequency (the number of times something happens) relates to a class interval (a range of numbers). **A histogram is useful when looking at how many times something happens**. It is useful to look at monthly or yearly temperatures, at test scores and groups of people based on age.

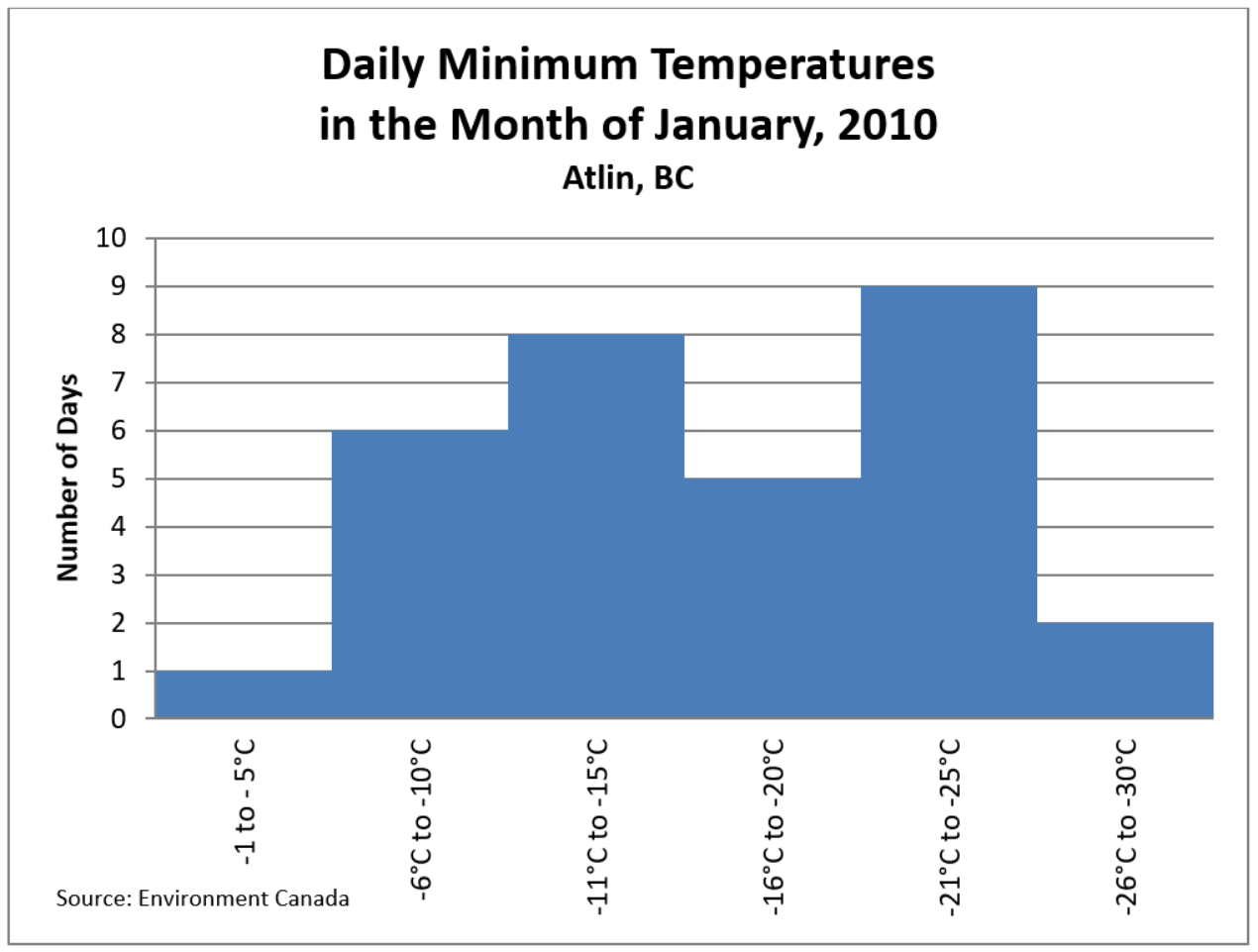
In the following graph, the height of each bar relates to how many days a temperature was between the listed temperatures in the horizontal axis.

This graph is created by counting how many days the temperatures were:

- between 0°C and −5°C (1 day)
- between −6°C and −10°C (6 days)
- between −11°C and −15°C (8 days)
- between −16°C and −20°C (5 days)
- between −21°C and −25°C (9 days)
- between −26°C and −30°C (2 days)

Then the information is put into graph form.

Graph 1



[Image Description]

A. How many degrees in temperature change is in each bar?
B. What is the source of the information?
C. Which temperature was the most common in the month of January?
D. Which community does this graph represent?
E. Which temperature was felt the least in Atlin in January, 2010?

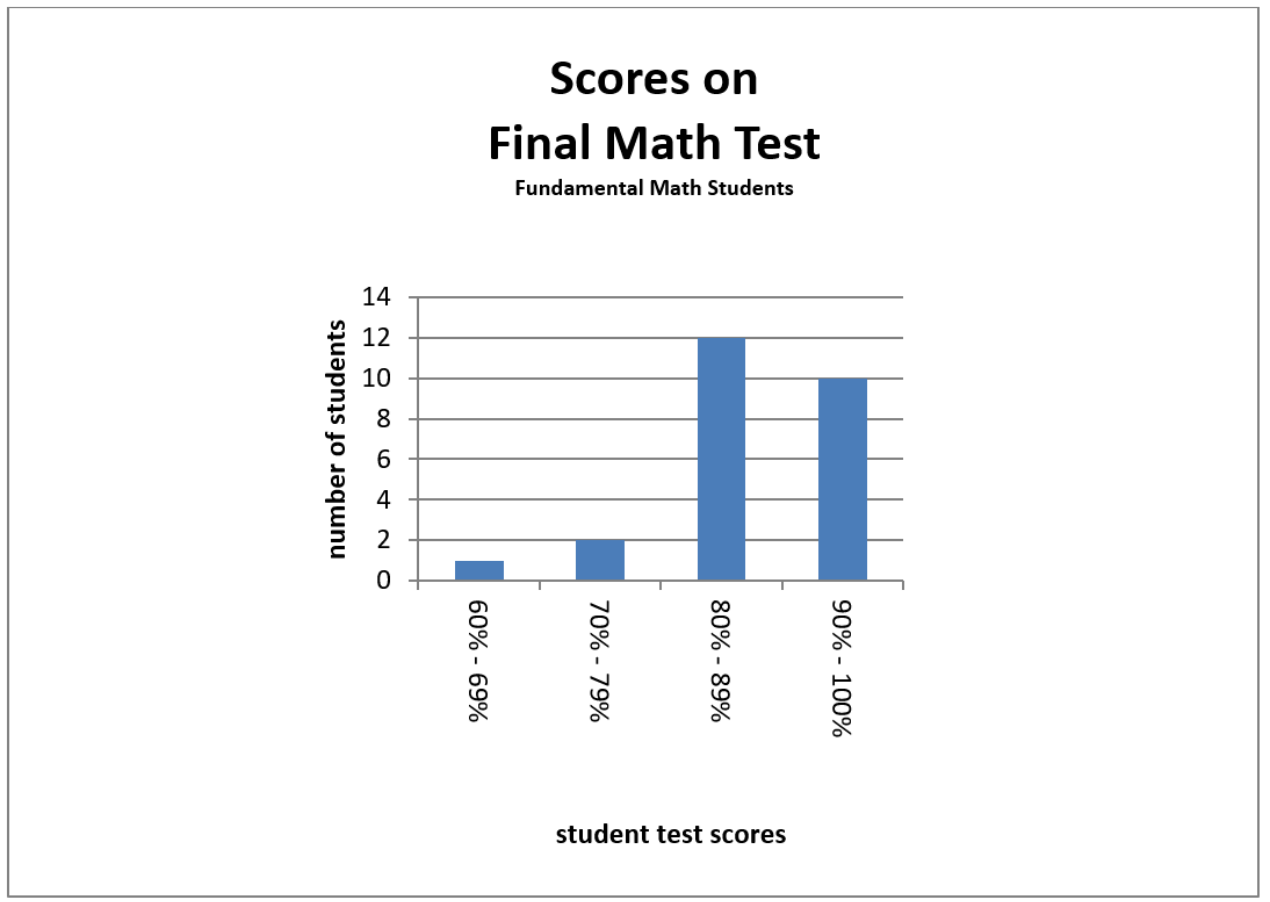
Answers to Graph 1

A. Five degrees
B. Environment Canada
C. −21°C to −25°C
D. Atlin, BC

E. −1°C to −5°C

Graph 2

In this Fundamentals Math course, students' marks were collected from the final test. The graph shows the results.



[Image Description]

A. How many students took the test?

B. How many students scored 80% to 89% on the test?

C. In order to pass, students must get 80% or over on the test. How many students will have to re-write the test?

D. How many more students got 80% to 89% than 90% to 100%?

Answers to Graph 2

A. 25
B. 12
C. 3
D. 2

Image Descriptions

Graph 1 (Histogram)

A histogram showing the daily minimum temperatures of Atlin, British Columbia in the month of January, 2010.

- The horizontal axis contains the following temperature ranges, in degrees Celsius: −1°C to −5°C, −6°C to −10°C, −11°C to −15°C, −16°C to −20°C, −21°C to −25°C, and −26°C to −30°C.
- The vertical axis is numbers of days, and has the numbers 0 through 10.

The histogram data is represented in the following table:

Daily Minimum Temperatures in the Month of January, 2010: Atlin, BC

Temperature Range in °C (Horizontal Axis)	**Number of Days (Vertical Axis)**
−1°C to −5°C	1
−6°C to −10°C	6
−11°C to −15°C	8
−16°C to −20°C	5
−21°C to −25°C	9
−26°C to −30°C	2
Source: Environment Canada	

[Return to Image]

Graph 2 (Histogram)

A histogram showing the scores on the final math test for Fundamentals Math students.

- The horizontal axis contains the following ranges of test scores, in percentages: 60%–69%. 70%–79%, 80%–89%, and 90%–99%.
- The vertical axis is numbers of students, and has the numbers 0 through 14 in increments of 2.

The histogram data is represented in the following table:

Scores on Final Math Test: Fundamentals Math Students

Test Score Range, in percentages (Horizontal Axis)	Number of Students (Vertical Axis)
60%–69%	1
70%–79%	2
80%–89%	12
90%–99%	10

[Return to Image]

Topic F: Tables

Tables are an everyday way of **organizing information, or one's own work**.

Graph 1

The following table is from the BC Ferries website: BC Ferries Current Conditions

It shows the departure times from each community.

Comox – Powell River (Little River-Westview) Crossing Time: 1 hour 20 minutes
Distance: 17 nautical miles

Leave Comox (Little River)	Leave Powell River (Westview)
6:30 am *Daily except Dec 25 and Jan 1*	8:10 am *Daily except Dec 25 and Jan 1*
10:10 am	12:00 pm
3:15 am	5:15 pm
7:15 am	8:45 pm

A. How many ferry runs go from Powell River to Comox each day?
B. On what days does the ferry not run at 6:30 am and 8:10 am?
C. How long is a crossing time?
D. When will the first ferry from Comox arrive in Powell River?
E. When will the last ferry from Powell River arrive in Comox?
F. How many nautical miles is covered by the ferry's course?

Answers to Graph 1

A. Four
B. Dec 25 and Jan 1
C. 1 hour and 20 minutes
D. 7:50 am
E. 10:05 pm
F. 17 nautical miles

Graph 2

A cereal recipe explains the quantities to use when making hot cereal.

Servings	Salt	Water	Cereal
1	¼ tsp	1.5 cups	¼ cups
4	1 tsp	6 cups	1 cup

A. How much salt should be put in for one serving?
B. How much water is to be used when making 4 servings of cereal?
C. How many cups of cereal should be used for making 1 serving of cereal?
D. To make ten servings of cereal, how much salt should be used?

Answers to Graph 1

A. ¼ tsp
B. 6 cups
C. ¼ cups
D. 2.5 tsp

Unit 5 Review: Statistics

Refer back to each lesson on graphs and explain when to use any three types of graph.

- Line Graph:
- Bar Graph:
- Picture Graph:
- Circle Graph:
- Histogram:
- Table:

TEST TIME!

Ask your instructor for the Practice Test for this unit.

Once you've done the Practice Test, you need to do the Unit 5 test.

Now that you have completed the whole book, you will need to do the Final test.

Please see your instructor for the Practice test and the Final Test.

Good luck!

Book 6 Review

You will now practice all the skills you learned in Book 6. You can use this as a review for your final test.

If you can't remember how to do a question, go back to the lesson on this topic to refresh your memory. The unit and topic for where each question came from is listed next to the question.

> **Example: *1A*** means Unit 1, Topic A

Unit 1

1-A

1. **Write the ratios asked for.**
    a. Lillian biked for 6 hours, and covered a total of 45 km. What is the ratio of kilometers to hours?
    b. Nine hundred cars were lined up at the ferry terminal. 300 hundred cars got on the next sailing. Write a ratio of how many cars were left behind to how many cars got on the first sailing.

1-B

2. **Simplify these ratios.**
    a. $9 : 12$
    b. $50 : 5$
    c. $56 : 7$
    d. $100 : 120$

3. **Write the following ratios as rates.**
    a. 110 kilometres to 2 hours
    b. 9 cups of flour to 3 tablespoons of yeast
    c. 240,000 people to 300 square kilometers

1-C

4. **Solve these proportions.**
    a. $1 : 3 = N : 12$
    b. $25 : N = 20 : 4$
    c. $N : 49 = 14 : 98$
    d. $4\frac{1}{2} : 6 = N : 3.6$
    e. The dose for cough syrup is 20 millilitres for each 100 pounds of body weight. How much should be given to a 34 pound child? Round to the nearest millilitre.

Unit 2

2-A

5. **Write these percents using numerals and the percent sign.**
    a. Seventy-two percent
    b. Three-fourths percent
    c. One hundred two percent

6. **Write these percents in words.**
    a. 12%
    b. $\frac{1}{5}\%$

7. **Change the percents to equivalent decimals.**
    a. 17%
    b. $98\frac{1}{2}\%$
    c. $\frac{1}{3}\%$

8. **Write the decimals as percents.**
    a. 0.45
    b. 4.75
    c. 0.099

9. **Change each percent to an equivalent common fraction. Put the fraction in lowest terms.**
    a. $33\frac{1}{3}\%$
    b. 14%
    c. 250%

10. **Write the percent equivalent**
    a. $\frac{1}{5}$
    b. $\frac{2}{3}$
    c. $\frac{1}{4}$

Unit 3

3-A

11. **Find the answers.**(Express percents rounded to the nearest tenth, money to the nearest cent and decimals to the nearest thousandth. Please show all your work. Use proportion.)
    a. 13% of $52 =$
    b. $\frac{9}{10}\%$ of $2,400$ is _____.
    c. What is 135% of $1,080$?

12. **Solve these problems. Be sure to show all your work.**
    a. Marianne is renovating her kitchen, and she is ordering everything from her local hardware store. She is getting a sink for $204.79, a dishwasher for $524.95, a counter for $949.99, flooring for $719.95, and a fridge for $579.49.
        i. Calculate the HST (12%).
        ii. Calculate the total cost, including the taxes.
    b. Shane sold a home for $340,500.00 for a client. He earned 6% commission. How much money did Shane make?
    c. A love seat is originally priced at $904.00, it is offered at 45% off. What is the discount price?
    d. Calculate the total cost in Canadian dollars of this purchase made in the United

States. Assume $1.00 Canadian = $0.92 U.S.
Clothes– Total price in $US = $317.98.

i. Price in Canadian dollars
ii. Duty at 13.5%
iii. Total of Canadian value + duty
iv. HST (12%) on Canadian value + duty
v. Total cost in Canadian dollars

Unit 4

4-A

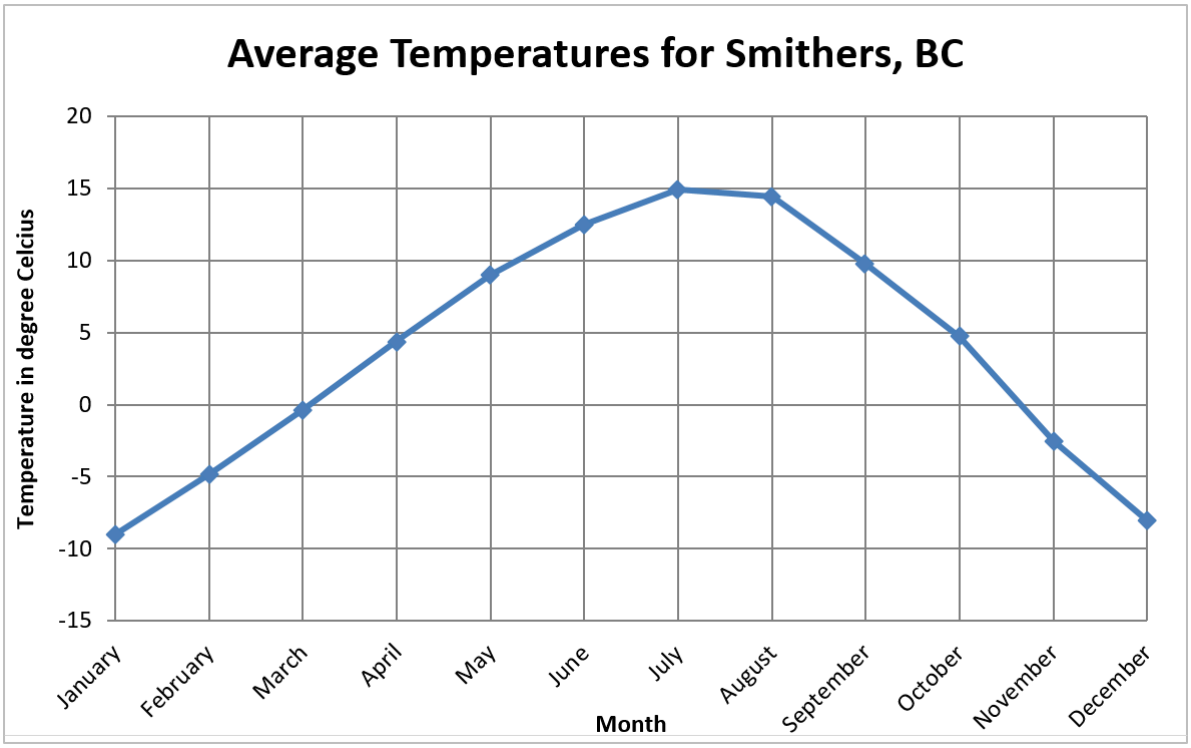
13. **Find the answers.**
    a. 34 is what percent of 85?
    b. What % of 150 is 114?
    c. 33⅓% of what number is 60?
    d. 32 is 20% of what number?
    e. 75% of what number is 675?
    f. 3.75 is 1¼% of?

14. **Solve these problems. Be sure to show all your work.**
    a. A printer is priced at $399. It is marked 30% off.
        i. What is the sale price of the printer?
        ii. What is the cost of this printer with HST (12%)?
    b. The Vancouver Fire and Rescue Service has 797 uniformed personnel, about 0.8% are women. About how many women are in uniform in the Vancouver Fire and Rescue Service?
    c. Jake is a computer salesperson. He receives a monthly salary of $1,055 plus 15% on all his sales over $5,500. What was his total monthly earnings if his sales were $12,400 in one month?
    d. The local ski hill sold 3,800 season's passes in 2009. The 2010 sales are down 10.5%. Find the number of season's passes sold in 2010.

Unit 5

5-A

15. **Line graph.**



[Image Description]

a. Which month has the highest temperature in Smithers?

b. Which month has the lowest temperature in Smithers?

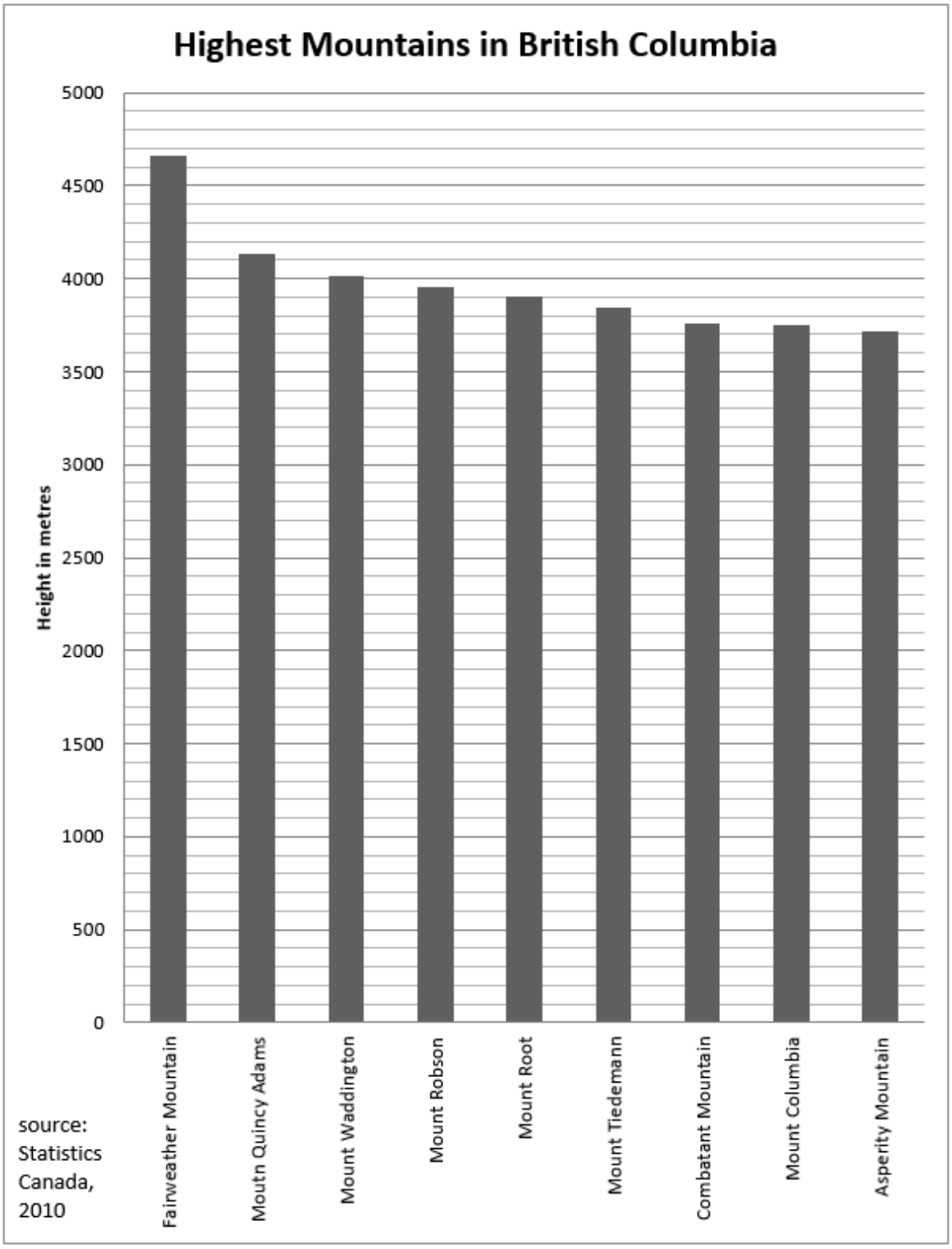
c. Between the months of January to July, is there an increase or decrease in temperature?

d. What is the difference between the monthly temperature for August and the monthly temperature for October?

e. What is the trend of the temperature in Smithers?

5-B

16. **Bar graph.**



[Image Description]

a. What is the height of the tallest mountain in BC?

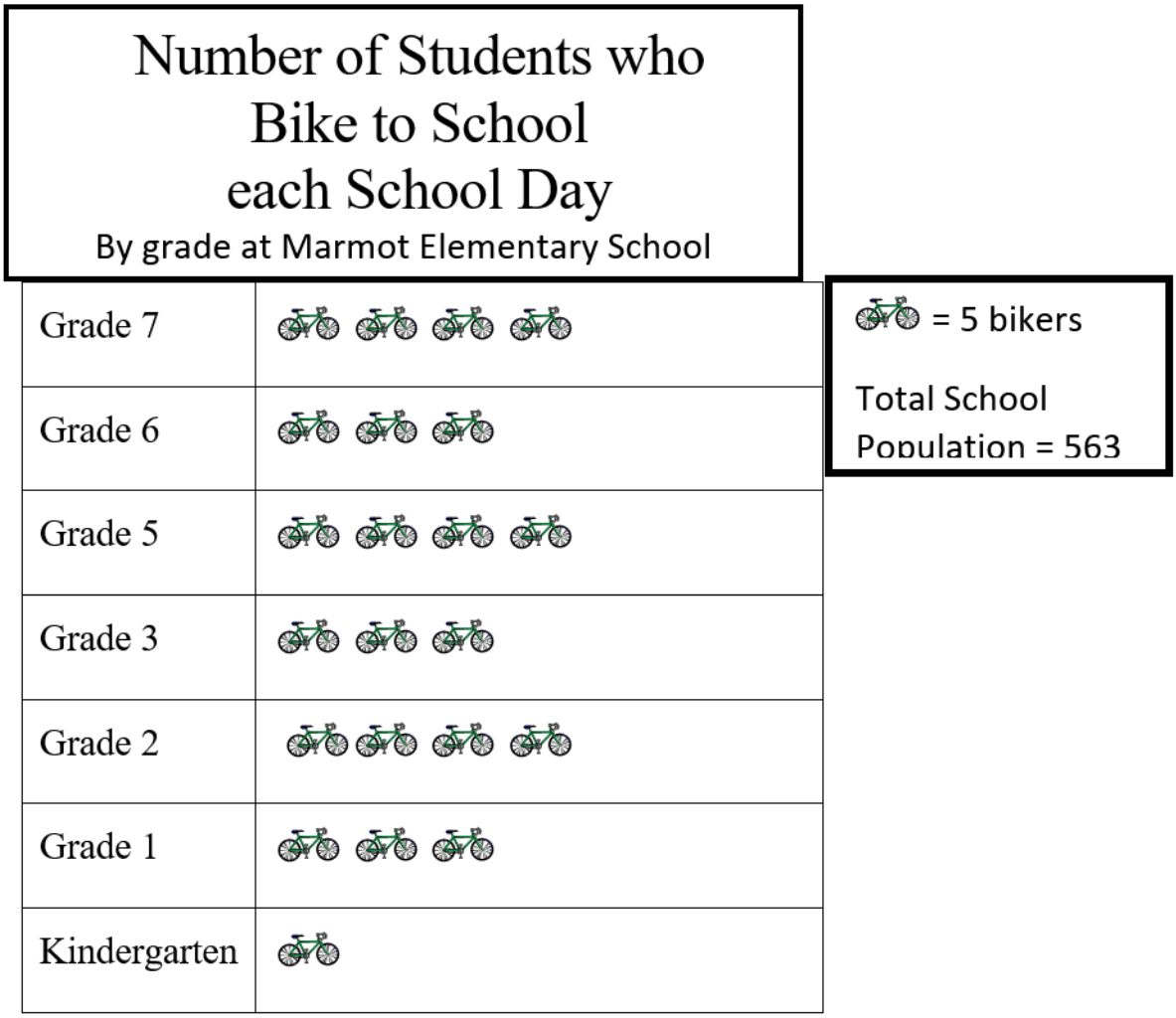
b. How many mountains are over 4,000 metres and under 5,000 metres in height?

c. Which two mountains in this chart are very similar in height?

d. What is the difference (approximately) in height between Fairweather Mountain and Asperity Mountain?

5-C

17. **Picture graph.**



[Image Description]

a. How many students in grade 2 ride bikes to school?

b. Which classes have the most bikers?

c. Which class has the least bikers?

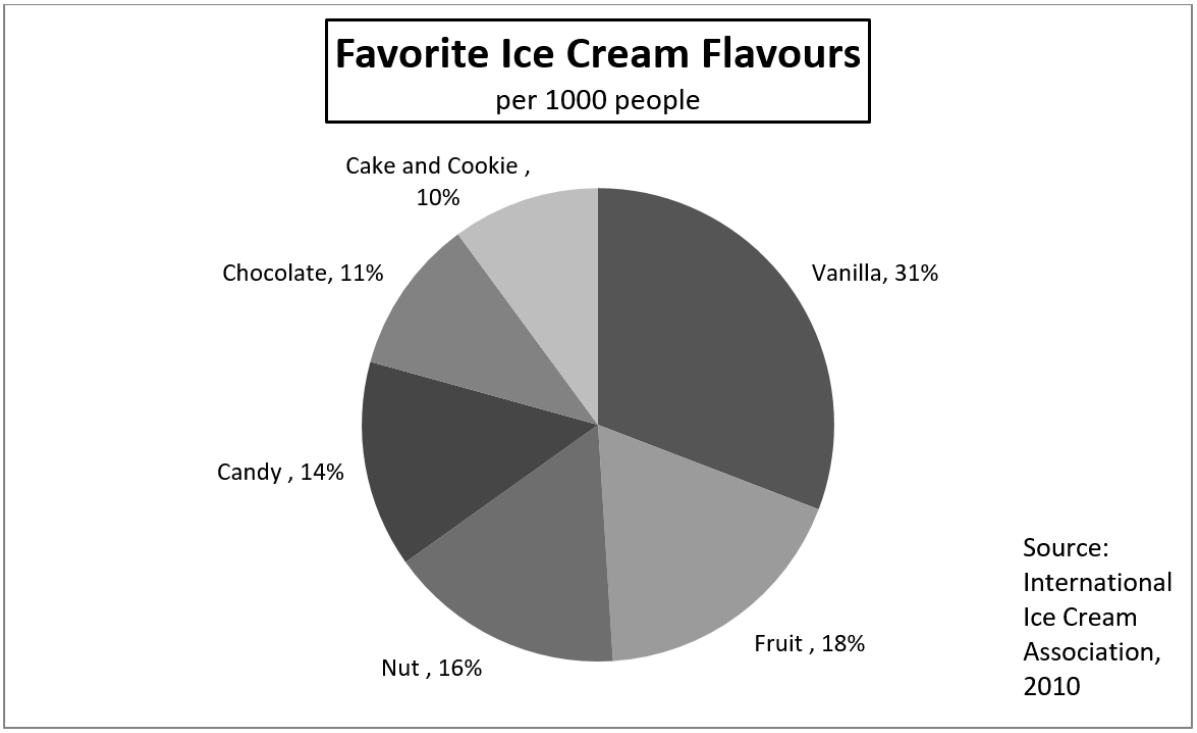
d. How many more bikers are in grade 7 than kindergarten?

e. How many students bike in total?

f. What percent of the school bikes each day?

5-D

18. **Circle graph.**



[Image Description]

a. Which is the most favourite ice cream flavour?
b. Which is the least liked ice cream flavour?
c. How many people (out of 1,000) like fruit-flavoured ice cream?
d. What percentage of people like vanilla over chocolate?

Answers to Book 6 Review

1. a. $45 : 6$
   b. $600 : 300$
2. a. $3 : 4$
   b. $10 : 1$

c. $8:1$

d. $5:6$

3. a. 55 km/hr or $55\text{km}:1\text{hr}$

b. 3 cups of flour to 1 tbsp yeast

c. 800 people to 1 square km

4. a. 4

b. 5

c. 7

d. 2.7

e. 7 mL

5. a. 72%

b. $\frac{3}{4}\%$

c. 102%

6. a. Twelve percent

b. One-fifth percent

7. a. 0.17

b. 0.985

c. $0.00\overline{3}$

8. a. 45%

b. 475%

c. 9.9%

9. a. $\frac{1}{3}$

b. $\frac{7}{50}$

c. $2\frac{1}{2}$

10. a. 20%

b. $66.\overline{6}\%$ or $66\frac{2}{3}\%$

c. 25%

11. a. 6.76
    b. 21.6
    c. $1,458$

12. a. i. $357.50
        ii. $3,336.67
    b. $20,430.00
    c. $497.20
    d. i. $345.63
        ii. $46.66
        iii. $392.29
        iv. $47.07
        v. $439.36

13. a. 40
    b. 76
    c. 180
    d. 160
    e. 900
    f. 300

14. a. i. $279.30
        ii. $312.82
    b. 6
    c. $2,090
    d. 3,401

15. a. July
    b. January
    c. Increase
    d. Approximately 9 degrees
    e. The temperature goes up from January to July, and goes down from July to December

16.
a. Approximately 4,650 metres
b. 3
c. Combatant Mountain and Mount Columbia
d. Approximately 950 metres

17.
a. 20
b. Grades 2, 5, 7
c. Kindergarten
d. 15
e. 110
f. Approximately 19.5%

18.
a. Vanilla
b. Cake and cookie
c. 180
d. 20%

Image Descriptions

Graph 1 (Line Graph)

A line graph displays the average temperature in Smithers, BC each month.

- The horizontal axis lists each month of the calendar year.
- The vertical axis is temperature in degrees Celsius, and contains the numbers -15 to 20 in increments of 5.

The line graph data is represented in the following table:

Average Temperatures for Smithers, BC

Month (Horizontal Axis)	Temperature in Degrees Celsius (Vertical Axis)
January	~−9
February	~−5
March	~0
April	~5
May	~9
June	~12.5
July	~15
August	~14
September	~10
October	~5
November	~−2.5
December	~−8

[Return to Image]

Graph 2 (Bar Graph)

A bar graph displays the height of the highest mountains in British Columbia in metres.

- The horizontal axis lists the following mountains: Fairweather Mountain, Mount Quincy Adams, Mount Waddington, Mount Robson, Mount Root, Mount Tiedemann, Combatant Mountain, Mount Columbia, and Asperity Mountain.
- The vertical axis is height in metres, and contains the numbers 5,000 in increments of 100.

The bar graph data is represented in the following table:

Highest Mountains in British Columbia

Mountain (Horizontal Axis)	**Height in metres (Vertical Axis)**
Fairweather Mountain	~4,650
Mount Quincy Adams	~4,100
Mount Waddington	~4,000
Mount Robson	~3,950
Mount Root	~3,900
Mount Tiedemann	~3,850
Combatant Mountain	~3,750
Mount Columbia	~3,750
Asperity Mountain	~3,700
Source: Statistics Canada, 2010	

[Return to Image]

Graph 3 (Picture Graph)

A picture graph displays the number of students at Marmot Elementary School who bike to school each day by grade.

- The vertical axis lists the following grades at Marmot Elementary School: Grade 7, Grade 6, Grade 5, Grade 3, Grade 2, Grade 1, and Kindergarten.
- The horizontal axis contains pictures of bicycles, with each bicycle representing 5 bikers according to the legend.
- The total school population is 563 students.

The picture graph data is represented in the following table:

Number of Students who Bike to School each School Day: By grade at Marmot Elementary School

Grade (Vertical Axis)	Pictures of Bikes (Horizontal Axis)
Grade 7	4
Grade 6	3
Grade 5	4
Grade 3	3
Grade 2	4
Grade 1	3
Kindergarten	1

[Return to Image]

Graph 4 (Circle Graph)

A circle graph displays the favourite ice cream flavours per 1,000 people.

- The entire graph represents all 1,000 people surveyed for their favourite ice cream flavour.
- Each part represents an ice cream flavour and its popularity as a percent of the whole. The flavours are (clockwise from top): Vanilla, Fruit, Nut, Candy, Chocolate, and Cake and Cookie.

The circle graph data is represented in the following table:

Favourite Ice Cream Flavours: per 1000 people[1]

Flavour	Percentage of Total
Vanilla	31%
Fruit	18%
Nut	16%
Candy	14%
Chocolate	11%
Cake and Cookie	10%

[Return to Image]

1. Source: Internal Ice Cream Association, 2010

Glossary

addends

The numbers to be added together in an addition question. In 3 + 5 = 8, the addends are 3 and 5.

axis

Any straight line used for measuring or as a reference.

balance

Balance has many meanings. In money matters, the balance is the amount left. It might be the amount left in a bank account (bank balance) or it might be the amount you still must pay on a bill (balance owing).

cancelled cheque

A cheque that has been cashed. The cheque is stamped, or cancelled, so it is no longer negotiable.

circumference

The distance around a circle; the perimeter of a circle.

commission

Salespeople may be paid a percentage of the money made in sales. The commission is part or all of their earnings.

common fractions

e.g., ⅔, ³⁄₇ , ⁴⁹⁄₅₀

cross multiply

In a proportion, multiply the numerator of the first fraction times the denominator of the second fraction. Then multiply the denominator of the first fraction times the numerator of the second fraction. In a true proportion, the products of the cross multiplication are equal.

denominator

The bottom number in a common fraction; tells into how many equal parts the whole thing has been divided.

diameter

The distance across a circle through its centre.

difference

The result of a subtraction question, the answer. Subtraction gives the difference between two numbers.

digit

Any of the ten numerals (0 to 9) are digits. This term comes from our ten fingers which are called digits. The numerals came to be called "digits" from the practice of counting on the fingers!

discount

An amount taken off the regular cost. If something is bought "at a discount" it is bought at less than the regular price.

divide

To separate into equal parts.

dividend

The number or quantity to be divided; what you start with before you divide.

divisor

The number of groups or the quantity into which a number (the dividend) is to be separated.

equal (=)

The same as

equation

A mathematical statement that two quantities are equal. An equation may use numerals with a letter to stand for an unknown quantity. 6 + Y = 9

equivalent

Equal in value; equivalent numbers (whole or fractions) can be used interchangeably; that is, they can be used instead of each other.

estimate

Make an approximate answer. Use the sign ≈ to mean approximately equal.

factors

The numbers or quantities that are multiplied together to form a given product. 5 × 2 = 10, so 5 and 2 are factors of 10.

fraction

Part of the whole; a quantity less than one unit.

horizontal

In a flat position, e.g. we are horizontal when we lie in a bed. A horizontal line goes across the page.

improper fraction

A common fraction with a value equal to or more than one.

infinite

Without end, without limit.

invert

To turn upside down.

like fractions

With the same denominators.

lowest terms

When the terms of a common fraction or ratio do not have a common factor (except 1), the fraction or ratio is in lowest terms (also called simplest form).

minuend

The first number in a subtraction question.

mixed decimal

A whole number and a decimal fraction. 1.75

mixed number

A whole number and a common fraction. 1 ¾

multiple

If a certain number is multiplied by another number, the product is a multiple of the numbers. Think of the multiplication tables. For example, 2, 4, 6, 8, 10, 12, 14... are multiples of 2.

multiplicand

The number to be multiplied.

multiplier

The number you multiply by.

negotiable

Something which can be cashed, that is, exchanged or traded as money.

numbers

Numbers represent the amount, the place in a sequence; *number* is the idea of quantity or order.

numerals

The digits 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 are also called numerals. These ten digits are combined to make infinite numerals. Digits are like letters, numerals are like words, and numbers are the meaning.

numerator

The top number in a common fraction; the numerator tells how many parts of the whole thing are being considered.

overdrawn

If the value of the cheques or money taken from a bank account is higher than the amount of money in the account, then the account is overdrawn. The account is "in the hole" or "in the red" are expressions sometimes used.

parallel

Two objects or lines side by side, never crossing and always the same distance from each other. Railway tracks are parallel, the lines on writing paper are parallel.

percent (%)

For every one hundred.

perimeter

The distance around the outside of a shape.

place value

We understand numbers by the way the digits (numerals) are arranged in relationship to each other and to the decimal point. Each position has a certain value. Our number system is a **decimal system**. The place value is based on **ten**.

prime number

A number that can only be divided evenly by itself and 1.

product

The result of a multiplying question, the answer.

proper fraction

A common fraction with a value less than one.

proportion

Generally, proportion is a way of comparing a part of something to the whole thing. E.g., his feet are small in proportion to his height. In mathematics, proportion is used to describe two or more ratios that are equivalent to each other.

quotient

The result of a division question; the quotient tells how many times one number is contained in the other.

radius

The distance from the centre of a circle to the outside of the circle.

ratio

The relationship between two or more quantities. E.g., the ratio of men to women in the armed forces is 10 to 3 (10:3)

reciprocal

A number, when multiplied by its reciprocal, equals 1. To find the reciprocal of a common fraction, invert it. $\frac{3}{5} \times \frac{5}{3} = 1$

reduce

Write a common fraction in lowest terms. Divide both terms by same factor.

remainder

The amount left when a divisor does not divide evenly into the dividend. The remainder must be less than the divisor.

sign

In mathematics, a symbol that tells what operation is to be performed or what the relationship is between the numbers.

\+ plus, means to add
− minus, means to subtract
× multiplied by, "times"
÷ divided by, division
= equal, the same quantity as
≠ not equal
≈ approximately equal
< less than
> greater than
≤ less than or equal to
≥ greater than or equal to

simplify

See *reduce*.

subtrahend

The amount that is taken away in a subtraction question.

sum

The result of an addition question, the answer to an addition question.

symbol

A written or printed mark, letter, abbreviation etc. that stands for something else.

term

a) A definite period of time, such as a school term or the term of a loan.

b) Conditions of a contract; the terms of the agreement.

c) In mathematics, the quantities in a fraction and in a ratio are called the *terms* of the fraction or

the *terms* of the ratio. In an algebra equation, the quantities connected by a + or − sign are also called terms.

total

The amount altogether.

transaction

One piece of business. A transaction often involves money. When you pay a bill, take money from the bank or write a cheque, you have made a transaction.

unit

Any fixed quantity, amount, distance or measure that is used as a standard. In mathematics, always identify the unit with which you are working. E.g., 3 km, 4 cups, 12 people, $76, 70 books, 545 g.

unit price

The price for a set amount. E.g., price per litre, price per gram.

unlike fractions

Fractions which have different denominators.

vertical

In an up and down position, e.g., we are vertical when we are standing up. On a page, a vertical line is shown from the top to the bottom of the page.

Acknowledgements - 1st Edition

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Versioning History

This page provides a record of edits and changes made to this book since its initial publication. Whenever edits or updates are made in the text, we provide a record and description of those changes here. If the change is minor, the version number increases by 0.01. If the edits involve substantial updates, the version number increases to the next full number.

The files posted by this book always reflect the most recent version. If you find an error in this book, please fill out the Report an Error form.

Version	Date	Change	Details
1.00	2014	Book published	
1.01	January 2016	Book updated.	
2.00	February 17, 2022	2nd Edition published.	
2.01	January 26, 2023	Minor edits for consistency of ALF Math series.	• Created a “How to Deal with Math Anxiety” front matter section, which is now standardized across all ALF Math books. • Added acknowledgments for the 1st edition to the back matter.